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FKAAS

Semester 1 2016/2017

Structural Analysis

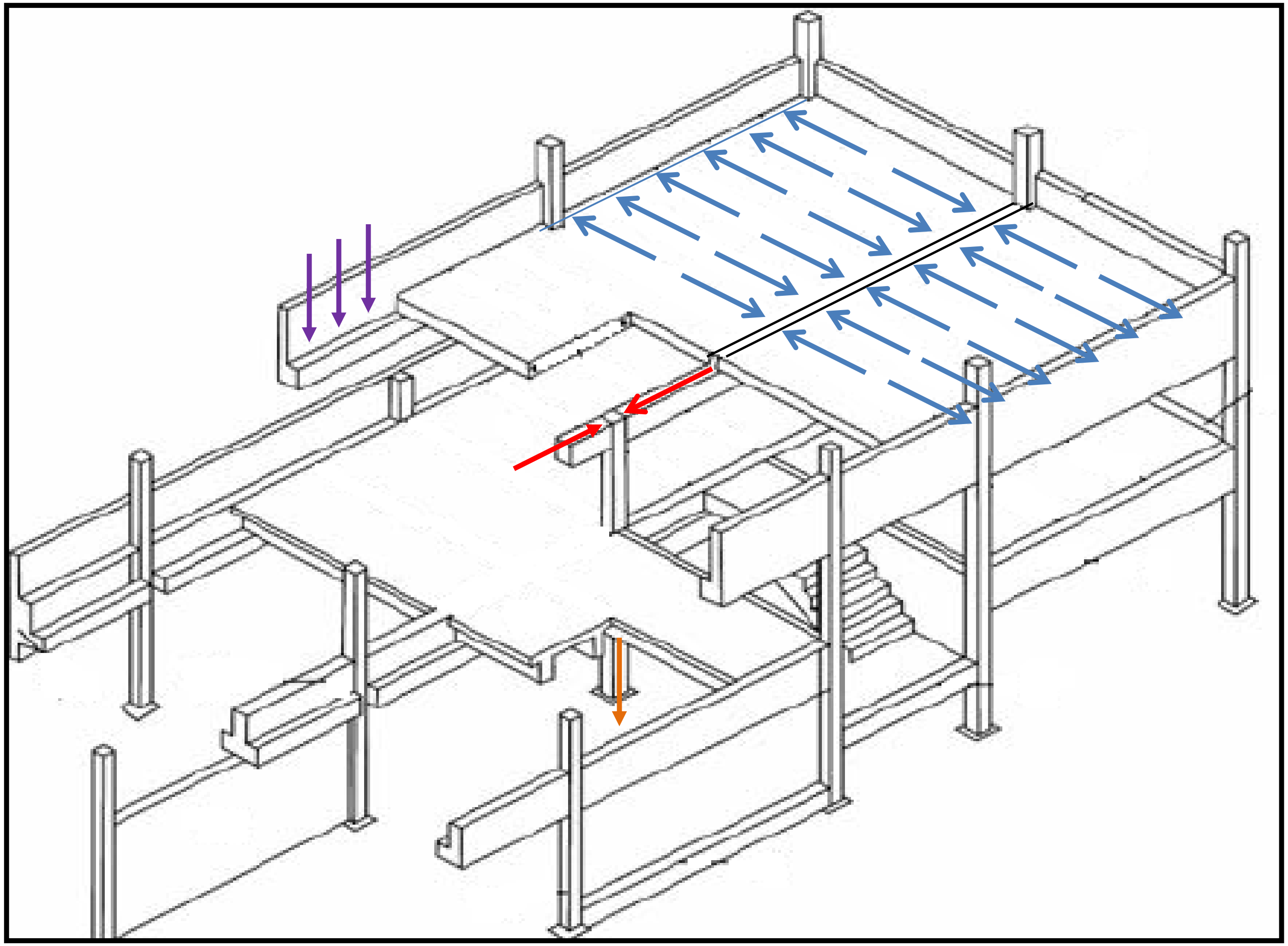
Department of Structures and Material Engineering
Faculty of Civil and Environmental Engineering
University Tun Hussein Onn Malaysia

With Wisdom We Explore

Introduction

- The primary purpose of structural analysis is to establish the distribution of internal forces and moments over the whole part of a structure and to identify the critical design conditions at all sections.
- The type of analysis should appropriate to the problem being considered.
- The following approach can be used: **linear elastic analysis**, **linear elastic analysis with limited redistribution**, and **plastic analysis**.
- Linear elastic analysis can be carried out by assuming that the cross section is uncracked (i.e. concrete section properties), involves linear stress-strain relationships and mean value of elastic modulus.
- Plastic analysis is desired in the design consideration, however, this approach requires advanced solutions.

Introduction



Distribution of actions on slab

↓

Action on slab and wall transfer to beam

↓

Axial load on column

↓

Distribute to foundation

Structural Layout

- Before any analysis and design can be conducted, structural layout (key-plan) must be produced.
- Structural layout planning is always started from the lowest floor. Step to produce structural layout:
  - Study and understand the architectural drawings (floor plans, elevations, cross sections, isometric view, specific details and so on).
  - Identify location and orientation of columns.
  - Identify location and position of beams.
  - Sketch the structural plans.
- For a simple layout, structural layout can be sketched on the architectural drawing by using colour pencil.
- For a complex layout, structural layout can be sketched on the butter paper by tracing from the architectural drawing.

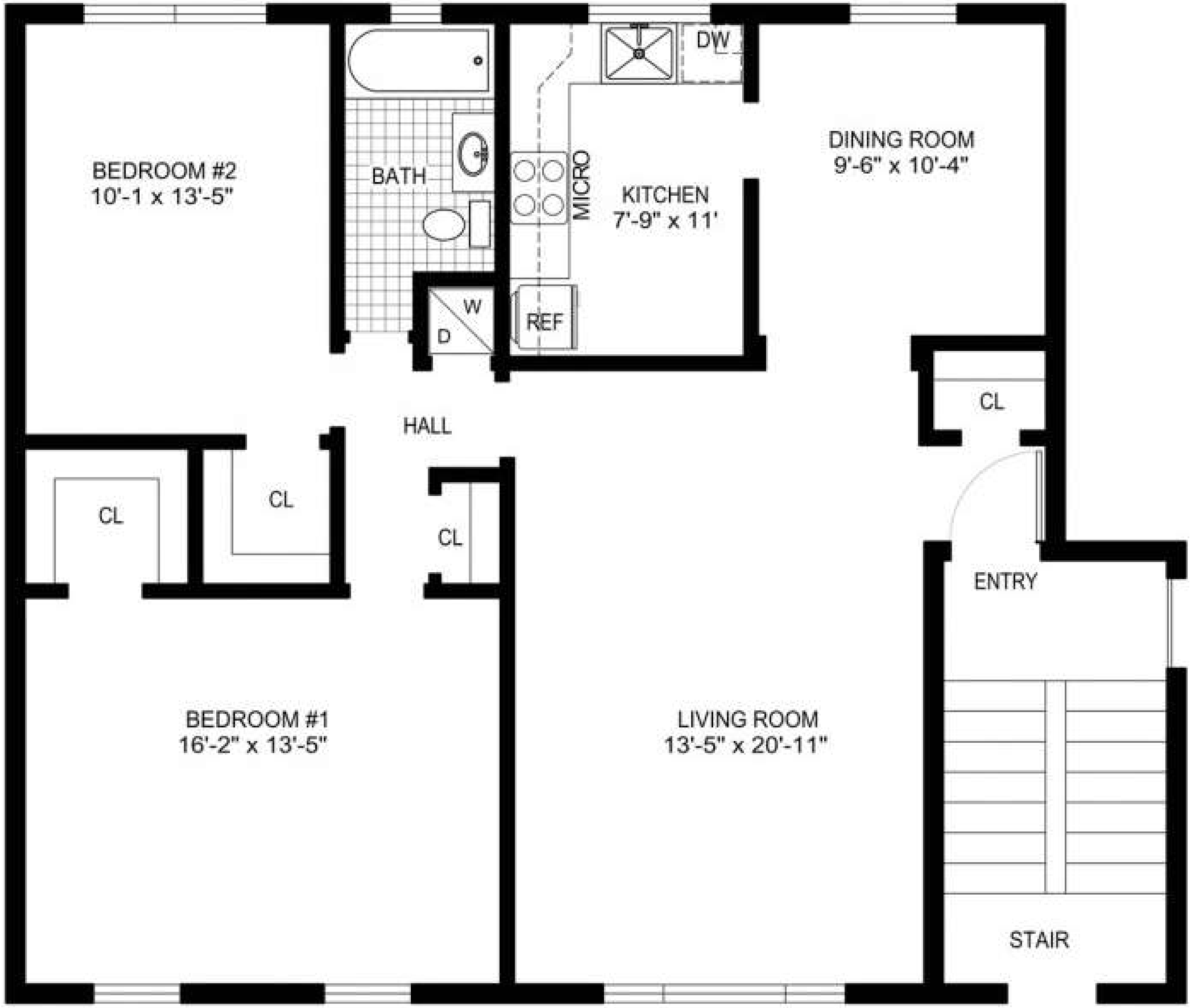
Structural Layout

- Location, orientation and dimension of columns:
  - Some are stated in the architectural drawings.
  - At the corner and intersection.
  - The distance between column and column is not too far and too close. Typically about 3m to 6m.
  - Flush with brickwall

- Location, position and dimension of beams
  - Location of brickwall, to brace the columns, to flush and brace the brickwall, to form spanning slab.
  - Dimension of beam is governed by thickness of brickwall, types of building, type of floor (ground or upper floor, upper floor with or without ceiling, head room), span and architectural drawing.

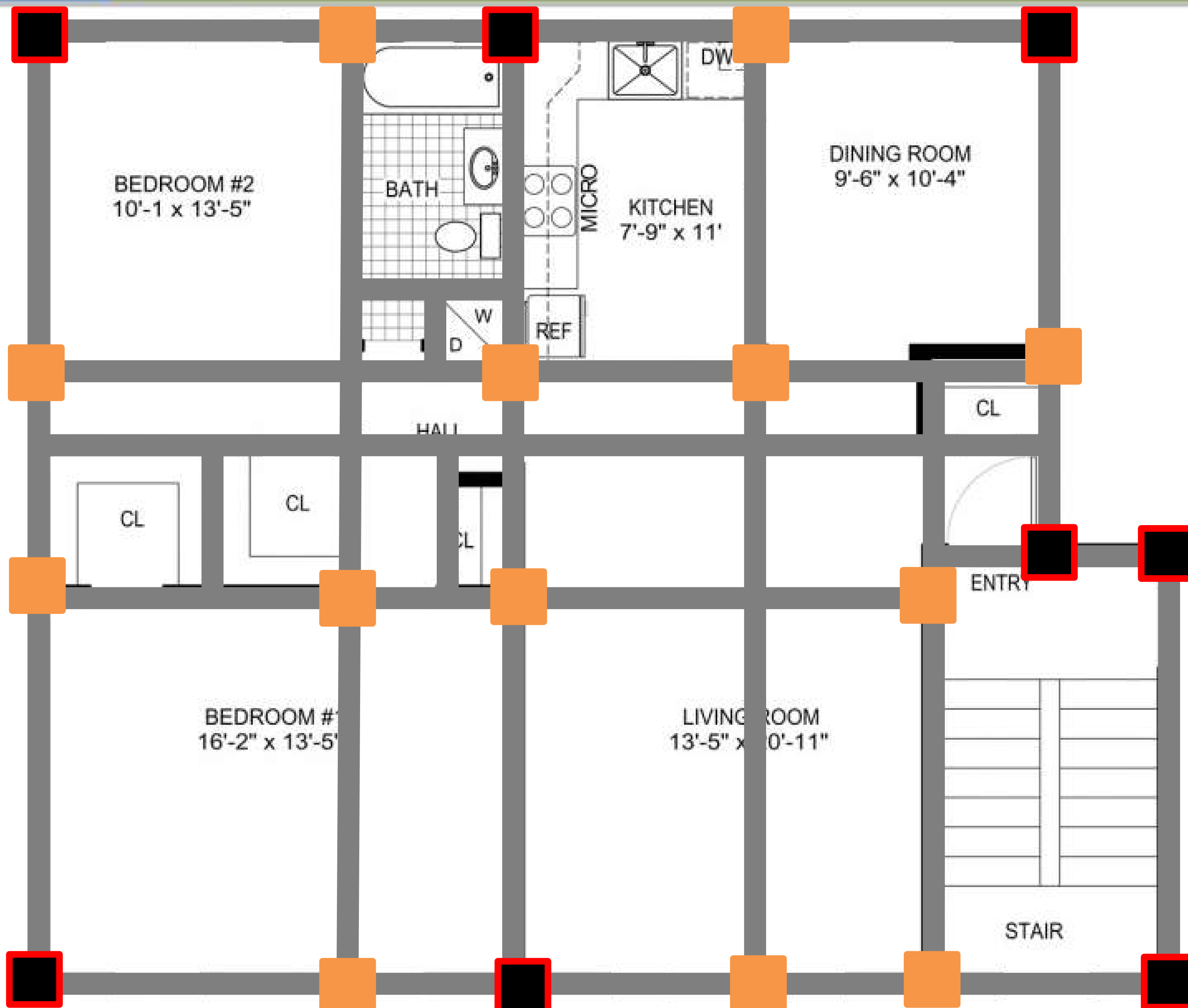
Structural Layout

Architectural plan



Structural Layout

Engineering Layout



Analysis of Actions

- Actions that applied on a beam may consist of beams self-weight, permanent and variable actions from slabs, actions from secondary beams and other structural or non-structural members supported by the beam.
- The distribution of slab actions on beams depends on the slab dimension, supporting system and boundary condition.
- It is important to determine the type of slab using following criteria:

$$\frac{L_y}{L_x} \geq 2$$ One-way slab

$$\frac{L_y}{L_x} < 2$$ Two-way slab

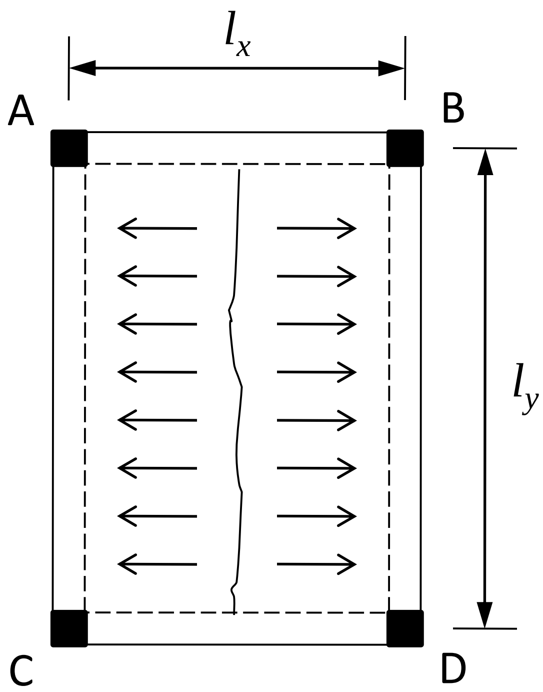
Analysis of Actions

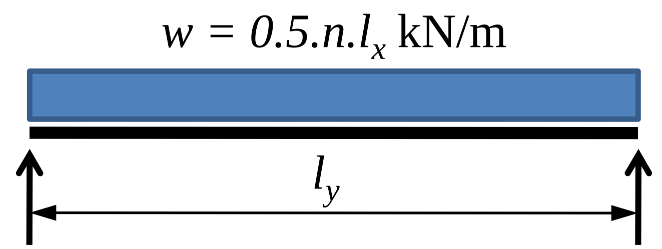
- Type of actions that must be considered:

Slab	✓ Permanent action: (i) Selfweight of slab, (ii) Finishes and services, and (iii) Ceiling ✓ Variable action (depend on function of floor)
Beam	✓ Permanent action: (i) Distribution from slab, (ii) Selfweight of beam, and (iii) Brickwall ✓ Variable action from slab
Column	✓ Permanent action: (i) Distribution from beam, and (ii) selfweight of column ✓ Variable action from beam
Foundation	✓ Permanent action: (i) Distribution from columns, and (ii) selfweight of footing ✓ Variable action from column

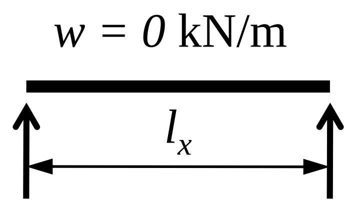
Analysis of Actions

- One-way spanning slab that supported by beams:





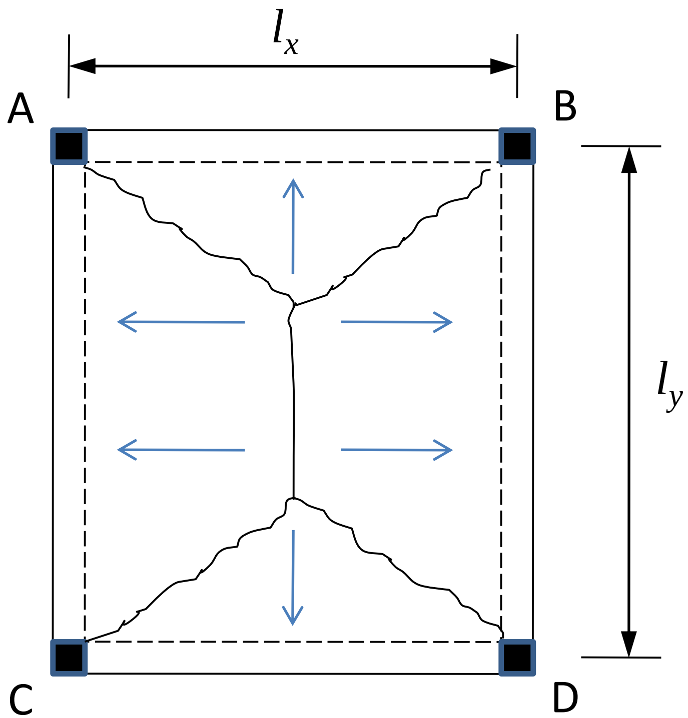
Beam AC and BD



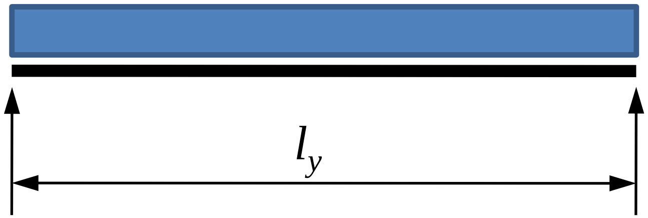
Beam AB and CD

Analysis of Actions

- Two-way slab panel freely supported along four edge:

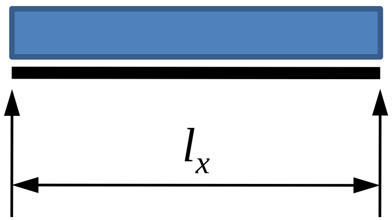


$$w = \frac{nl_x}{6}\left[3 - \left(\frac{l_x}{l_y}\right)^2\right] kN/m$$



Beam AC and BD

$$w = \frac{nl_x}{3} kN/m$$



Beam AB and CD

How about if $l_x = l_y$?

Analysis of Actions

- There are alternatives methods which consider various support conditions and slab continuity. The methods are (i). Slab shear coefficient from **Table 3.15 BS 8110**, (ii). Yield line analysis and (iii). Table 63 Reinforced Concrete Designer's Handbook by Reynold.

Table 3.15 — Shear force coefficient for uniformly loaded rectangular panels supported on four sides with provision for torsion at corners

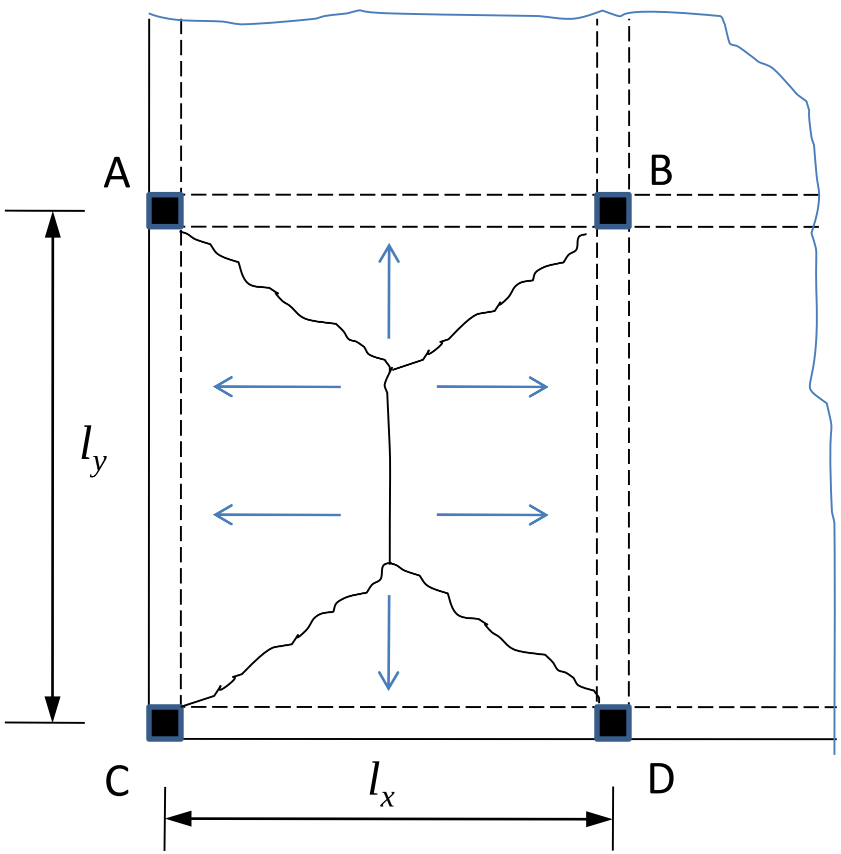
Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.75**	**2.0**	
Four edges continuous									
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	—	—	—	—	—	—	—	—	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	—

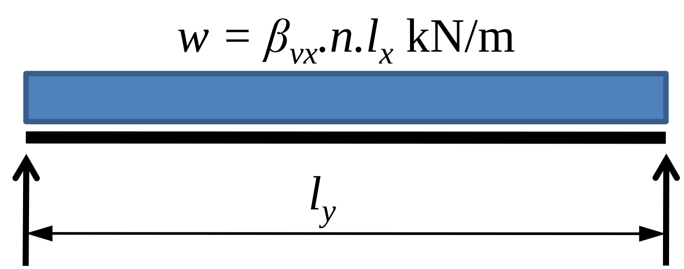
This table only can be used for two-way slabs.

The type of spanning slab (from 9 cases) must be identified first.

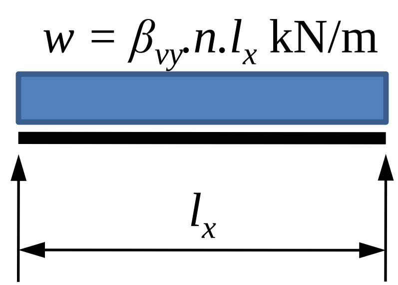
Analysis of Actions

- Two-way spanning slab:





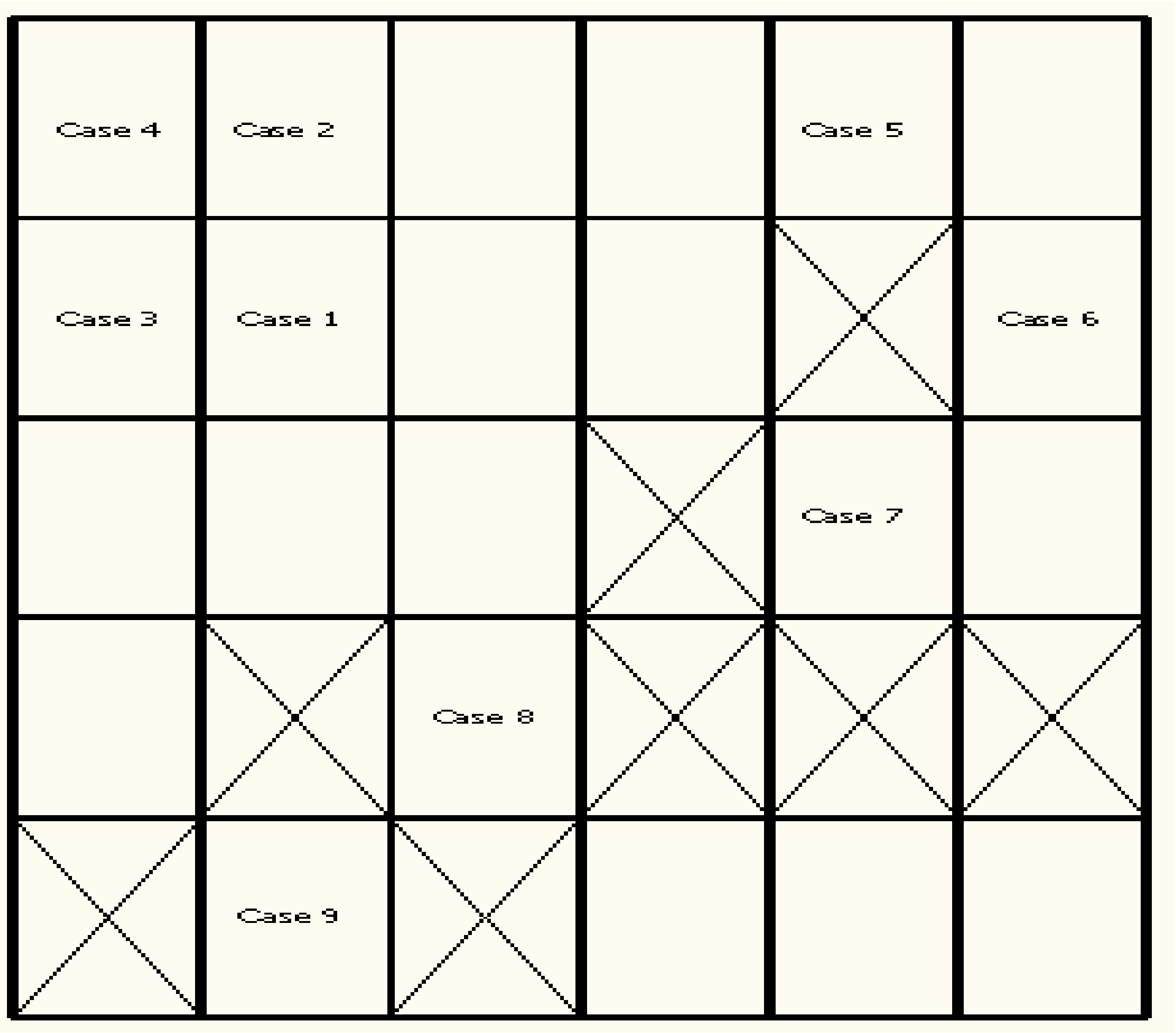
Beam AC



Beam CD

Analysis of Actions

9 cases of two-way restrained slab



Analysis of Actions

Table 3.15 — Shear force coefficient for uniformly loaded rectangular panels supported on four sides with provision for torsion at corners

	Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Case 1	**Four edges continuous**									
	Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
Case 2	**One short edge discontinuous**									
	Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
	Discontinuous edge	—	—	—	—	—	—	—	—	0.24
Case 3	**One long edge discontinuous**									
	Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
	Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	—
Case 4	**Two adjacent edges discontinuous**									
	Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
	Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
Case 5	**Two short edges discontinuous**									
	Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	—
	Discontinuous edge	—	—	—	—	—	—	—	—	0.26
Case 6	**Two long edges discontinuous**									
	Continuous edge	—	—	—	—	—	—	—	—	0.40
	Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	—
Case 7	**Three edges discontinuous (one long edge discontinuous)**									
	Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	—
	Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
Case 8	**Three edges discontinuous (one short edge discontinuous)**									
	Continuous edge	—	—	—	—	—	—	—	—	0.45
	Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
Case 9	**Four edges discontinuous**									
	Discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33

Example 1

- Determine the characteristic permanent and variable action on beam B/1-3.Given the following data: Unit weight of concrete = 25kN/m^3; Finishes, ceiling and services = 2.0kN/m^2;Variable action (all slabs) = 3.0kN/m^2.

Action on slab:

Selfweight = 0.15 x 25	= 3.75 kN/m^2
Finishes, ceiling and services	= 2.0 kN/m^2
Chac. Permanent action, G_k	= 5.75 kN/m^2
Chac. Variable action, Q_k	= 3.0 kN/m^2

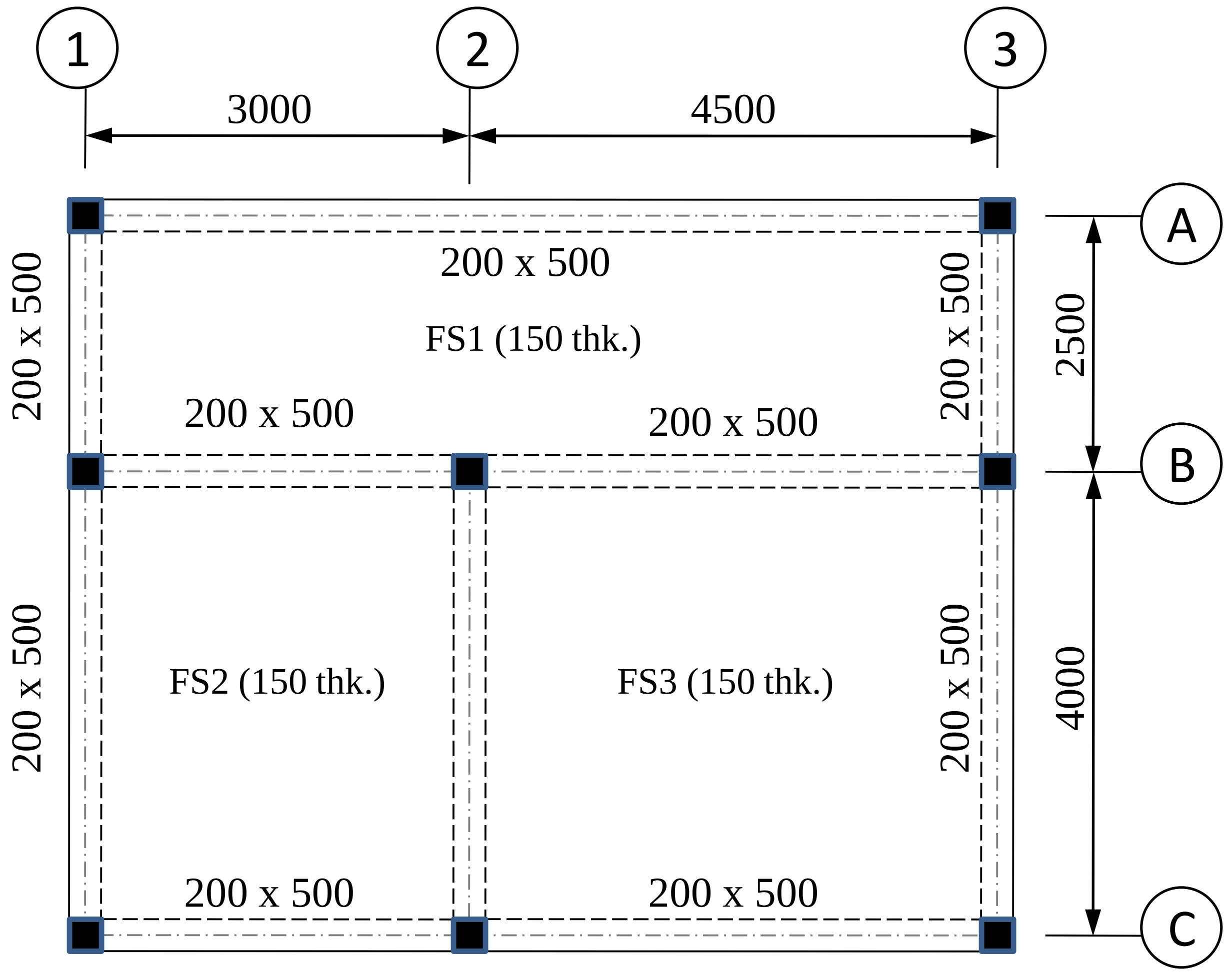
Distribution of actions

FS1 : l_y/l_x = 7.5 / 2.5 = 3 > 2.0, One-way slab

FS2 : l_y/l_x = 4.0 / 3.0 = 1.33 < 2.0, Two-way slab

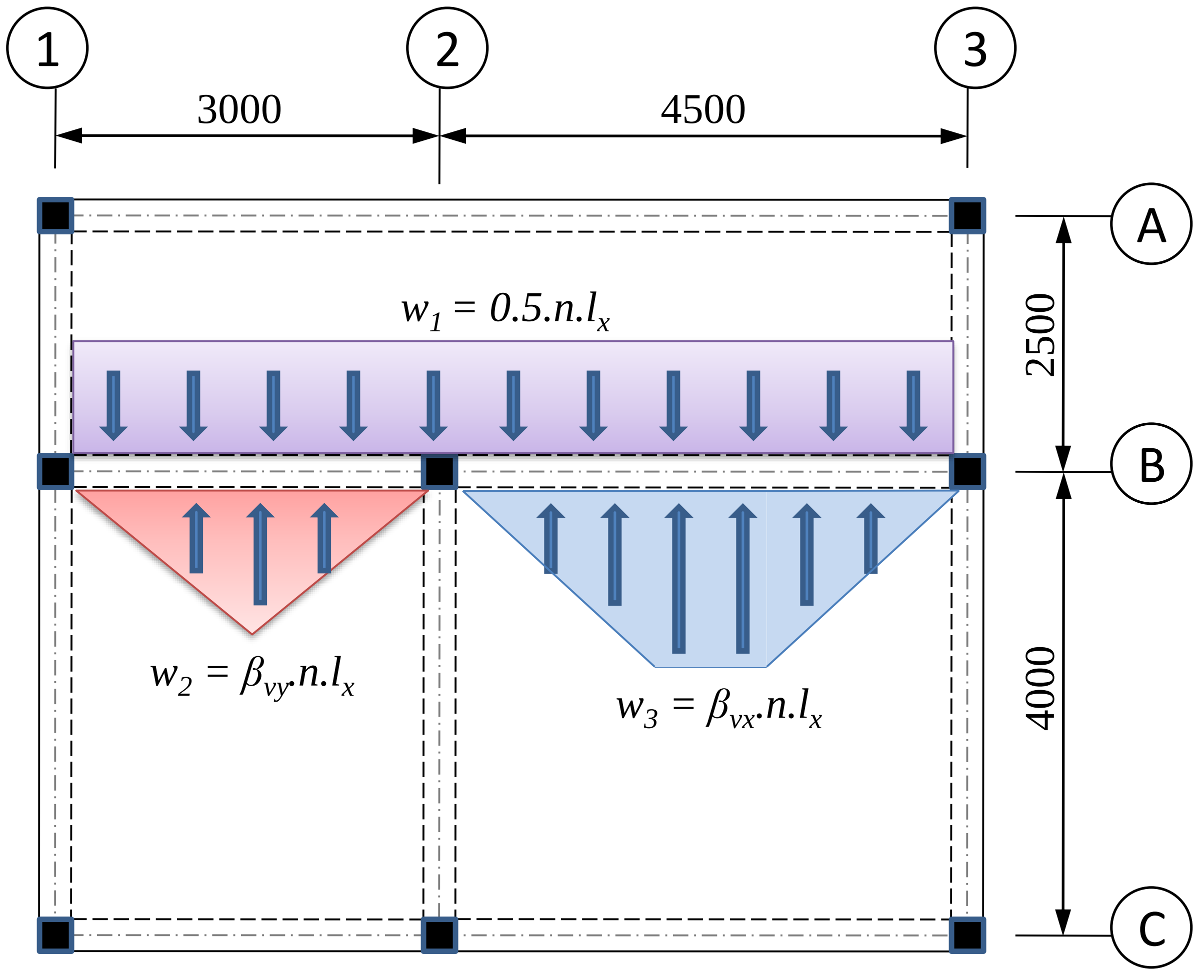
FS3 : l_y/l_x = 4.5 / 4.0 = 1.13 < 2.0, Two-way slab

Example 1



All unit in mm

Example 1



All unit in mm

Example 1

Action from slab:

w_1 G_k = 0.5 x 5.75 x 2.5 = 7.19 kN/m

w_1 Q_k = 0.5 x 3.00 x 2.5 = 3.75 kN/m

From Table 3.15: BS 8110: Part 1: 1997

Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Two adjacent edges discontinuous									
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26

w_2 G_k = 0.4 x 5.75 x 3.0 = 6.90 kN/m

w_2 Q_k = 0.4 x 3.00 x 3.0 = 3.60 kN/m

w_3 G_k = 0.44 x 5.75 x 4.0 = 10.12 kN/m

w_3 Q_k = 0.44 x 3.00 x 4.0 = 5.28 kN/m

Example 1

Actions on beam:

Beam selfweight = 0.20x(0.5–0.15)x25 = 1.75 kN/m

Span 1-2

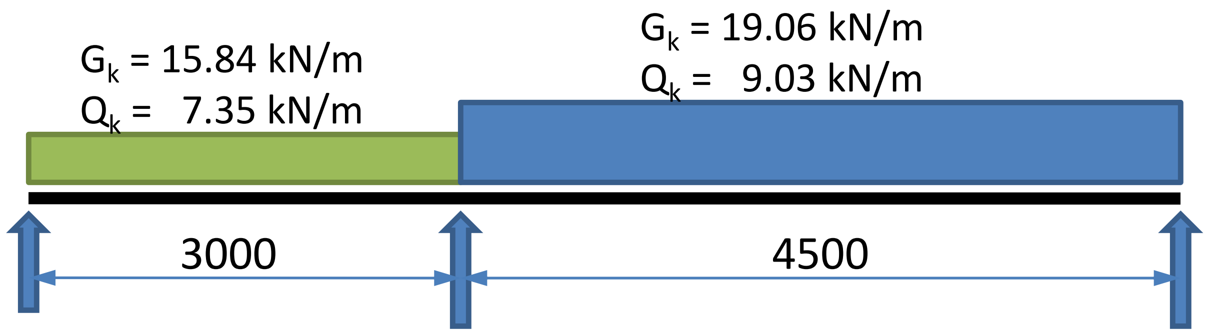
Permanent action, G_k = 7.19+6.90+1.75 = 15.84 kN/m

Variable action, Q_k = 3.75+3.60 = 7.35 kN/m

Span 2-3

Permanent action, G_k = 7.19+10.12+1.75 = 19.06 kN/m

Variable action, Q_k = 3.75+5.28 = 9.03 kN/m



Combination of Action

- "Combination of action" is specifically used for the definition of the magnitude of actions to be used when a limit state is under the influence of different actions.
- For continuous beam, "Load cases" is concerned with the arrangement of the variable actions to give the most unfavourable conditions or most critical responses.
- If there is only one variable actions (e.g. imposed load) in a combination, the magnitude of the actions can be obtained by multiplying with the appropriate factors.
- If there is more than one variable actions in combination, it is necessary to identify the leading action($Q_{k,1}$) and other accompanying actions ($Q_{k,i}$). The accompanying actions is always taken as the combination value.

Combination of Action

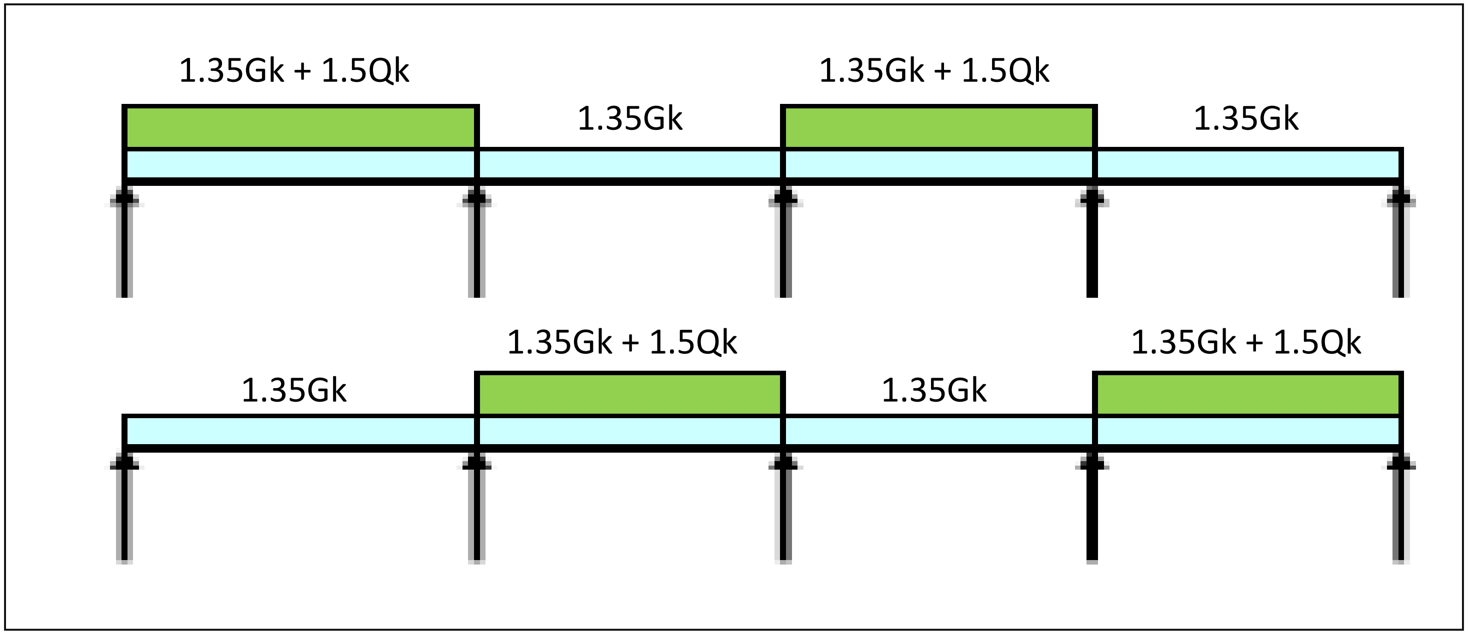
- In considering the combinations of actions, the relevant cases shall be considered to enable the critical design conditions to be established at all sections, within the structure or part of the structure considered.
- For simply supported beam, the analysis for bending and shear force can be carried out using statically determinate approach. For the ultimate limit state we need only consider the maximum load of $1.35G_k + 1.5Q_k$ on the span.
- For continuous beam, the following simplified load arrangements (based on National Annex) are recommended:
  - Load set 1: Alternate or adjacent spans loaded
  - Load set 2: All or alternate spans loaded

Combination of Action

- Load set 1: Alternate or adjacent spans loaded (**Section 5.1.3 : MS EN 1992-1-1**)
  - Alternate span carrying the design permanent and variable load (1.35*Gk* + *1.5Qk), other spans carrying only the design permanent loads (1.35Gk)*
  - Any two adjacent spans carrying the design permanent and variable loads (1.35*Gk* + *1.5Qk), all other spans carrying only the design permanent load (1.35Gk)*
- Load set 2: All or alternate spans loaded **(UK National Annex)**
  - All span carrying the design permanent and variable loads (1.35*Gk*+ *1.5Qk)*
  - Alternate span carrying the design permanent and variable load (1.35*Gk*+ *1.5Qk), other spans carrying only the design permanent loads (1.35Gk)*

Combination of Action

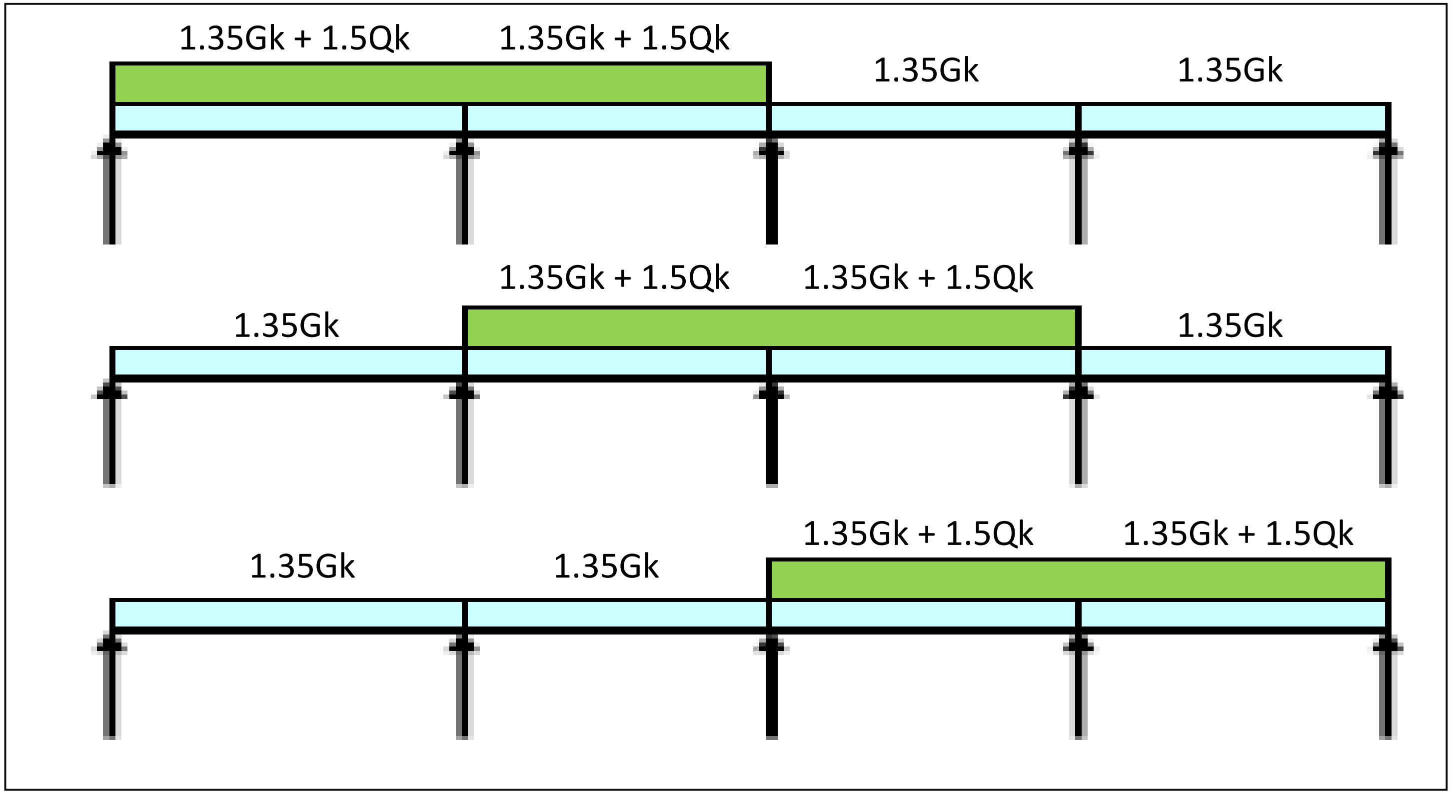
- Load set 1: Alternate spans loaded



Maximum action = 1.35Gk + 1.5Qk
Minimum action = 1.35Gk

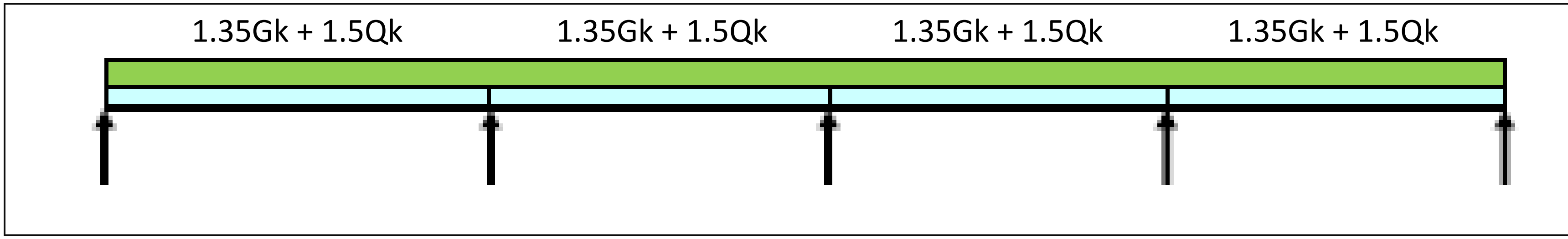
Combination of Action

- Load set 1: Adjacent Span Loaded

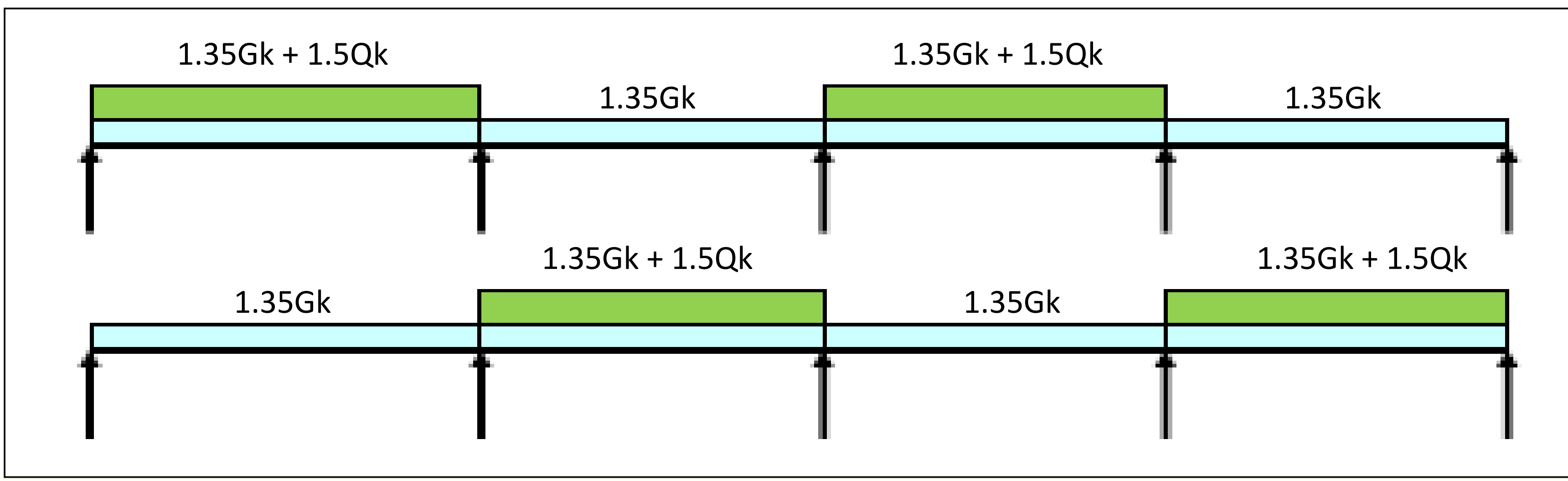


Combination of Action

- Load set 2: All span loaded



- Load set 2: Alternate span loaded



Moment and Shear Force

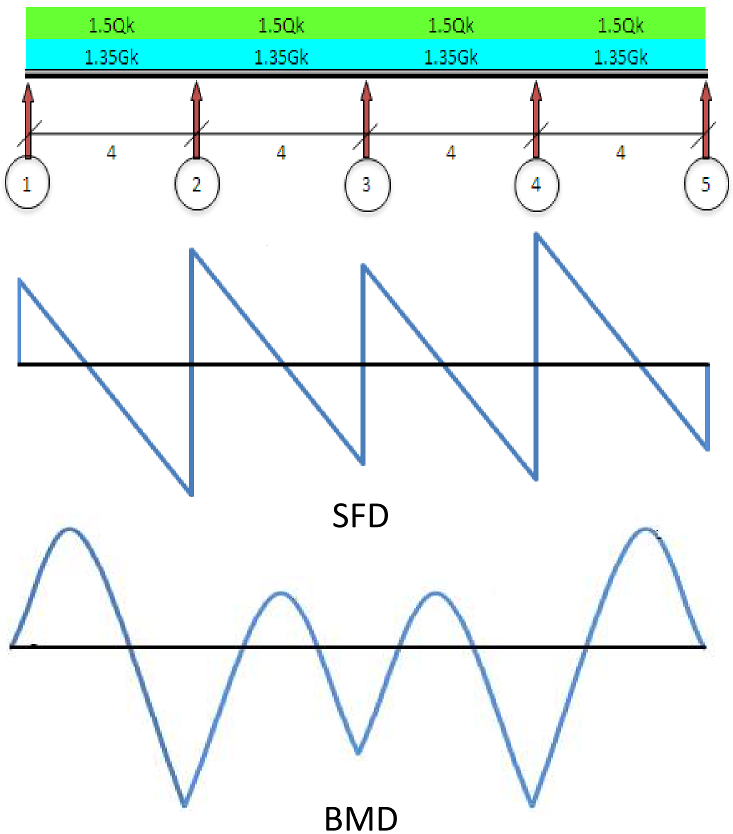
- The shear force and bending moment diagrams can be drawn for each of the load cases required in the patterns of loading.
- A composite diagram comprising a profile indicating the maximum values including all possible load cases can be drawn; this is known as an envelope.
- Three analysis methods may be used in order to obtain shear force and bending moment for design purposes. There are;

1. Elastic analysis using moment distribution method (Modified Stiffness Method)
2. Simplified method using shear and moment coefficient from Table 3.6: BS 8110: Part 1.
3. Using commercial analysis software such as Staad.Pro, Esteem, Ansys, Lusas, etc.

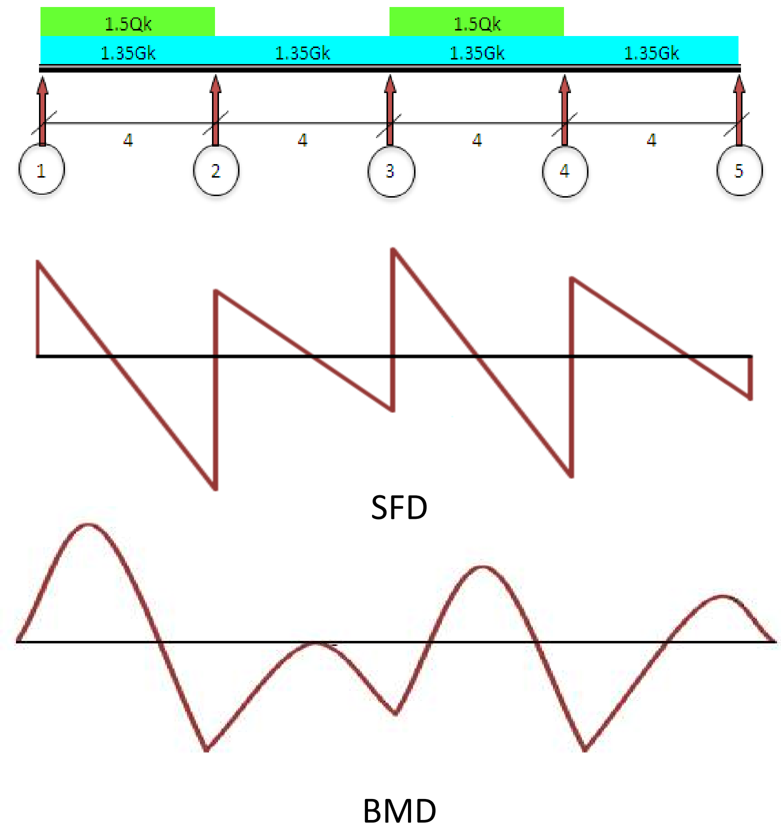
Moment and Shear Force

- Envelope moment and shear force:

Load Case 1



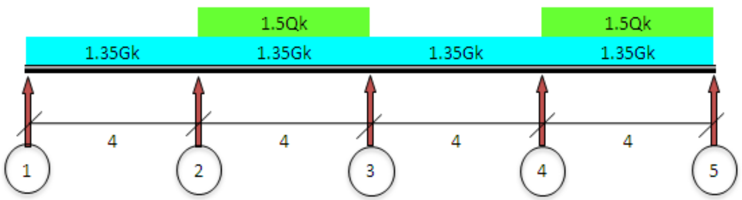
Load Case 2

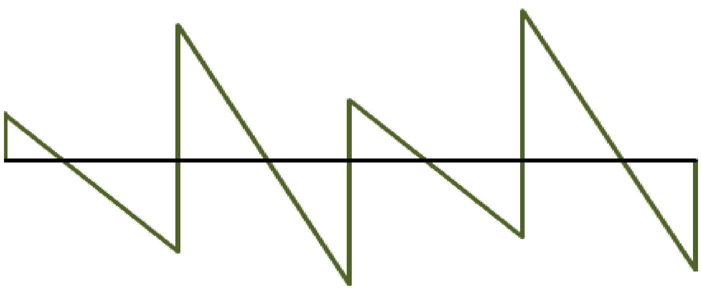


Moment and Shear Force

- Envelope moment and shear force:

Load Case 3





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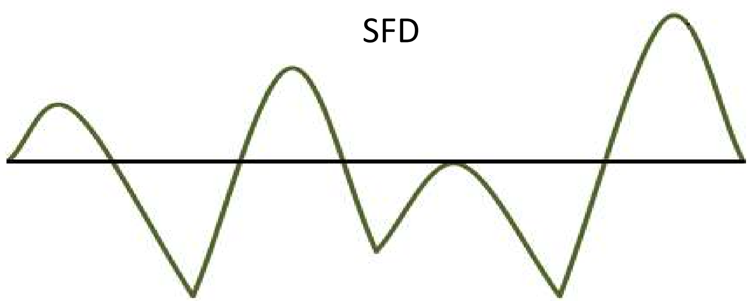

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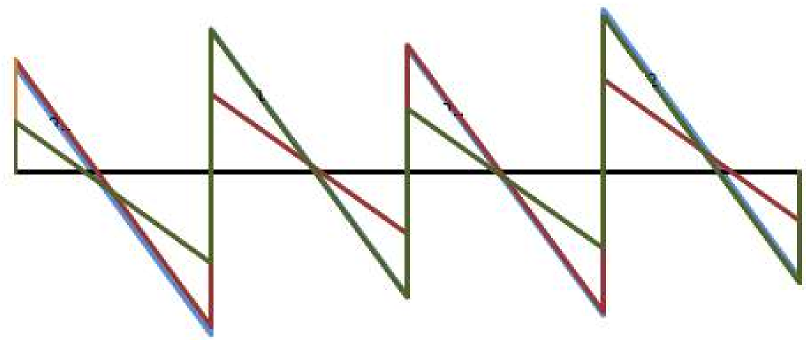
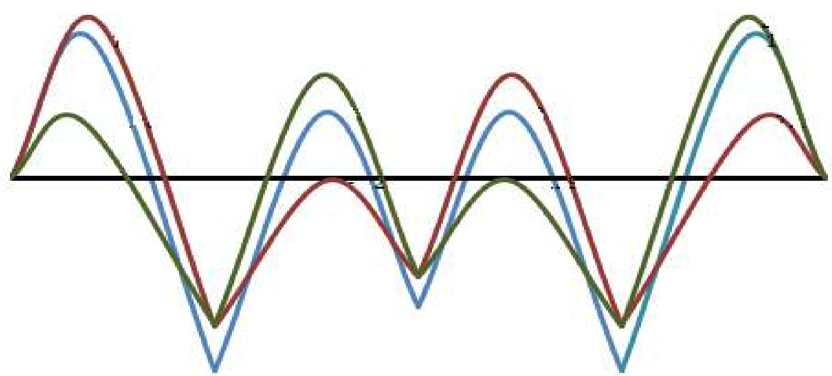
SHEAR FORCE DIAGRAM ENVELOPE



BENDING MOMENT DIAGRAM ENVELOPE



Moment Distribution Method

- Moment distribution method is only involving distribution moments to joint repetitively.
- The accuracy of moment distribution method is dependent to the number repeat which does and usually more than 5 repeat real enough. Right value will be acquired when no more moments that need distributed.
- In general the value is dependent to several factor as :
  - Fixed end moment - the moment at the fixed joints of a loaded member.
  - Carry over factor - the carry-over factor to a fixed end is always 0.5, otherwise it is zero.
  - Member stiffness factor (distribution factor) – need to be determined based on moment of inertia and stiffness.

Simplified Method

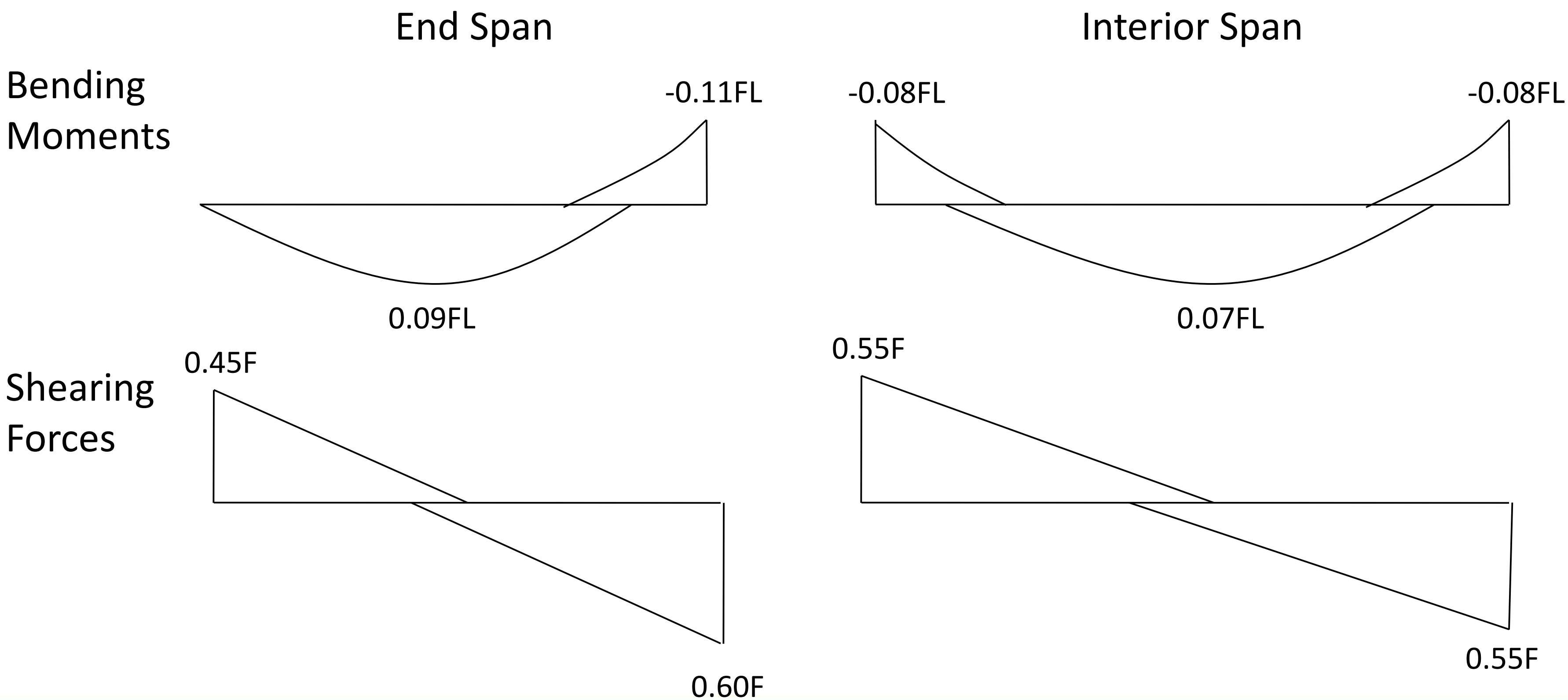
- The analysis using Moment Distribution Method is time consuming.
- Therefore, as a simplification, Cl. 3.4.3 BS 8110 (Table 3.5) can be used. This simplified method enables a conservative estimation of shear force and bending moment for continuous beam.
- However, there are conditions which must be satisfied:
  - The beams should be approximately equal span.
  - Variation in span length should not exceed 15% of the longest span.
  - The characteristic variable action, *Qk* may not exceed the characteristic permanent action, *Gk*.
  - Load should be substantially uniformly distributed over three or more spans.

Simplified Method

	At outer support	Near middle of end span	At first interior support	At middle of interior spans	At interior supports
Moment	0	$0.09Fl$	$-0.11Fl$	$0.07Fl$	$-0.08Fl$
Shear	$0.45F$	—	$0.6F$	—	$0.55F$

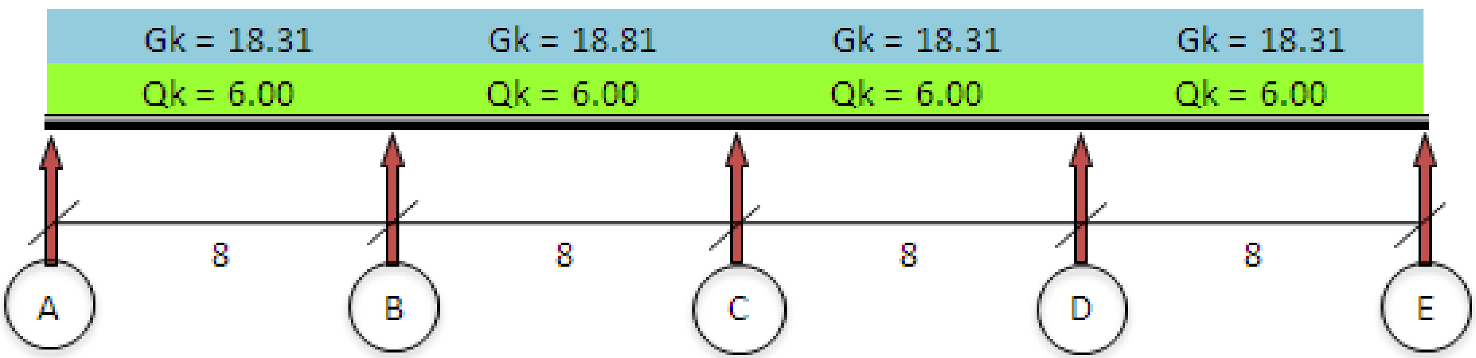
NOTE l is the effective span;
F is the total design ultimate load (1.35Gk + 1.5Qk)

No redistribution of the moments calculated from this table should be made.



Example 2

- By using simplified method, analyze the beam as shown below.

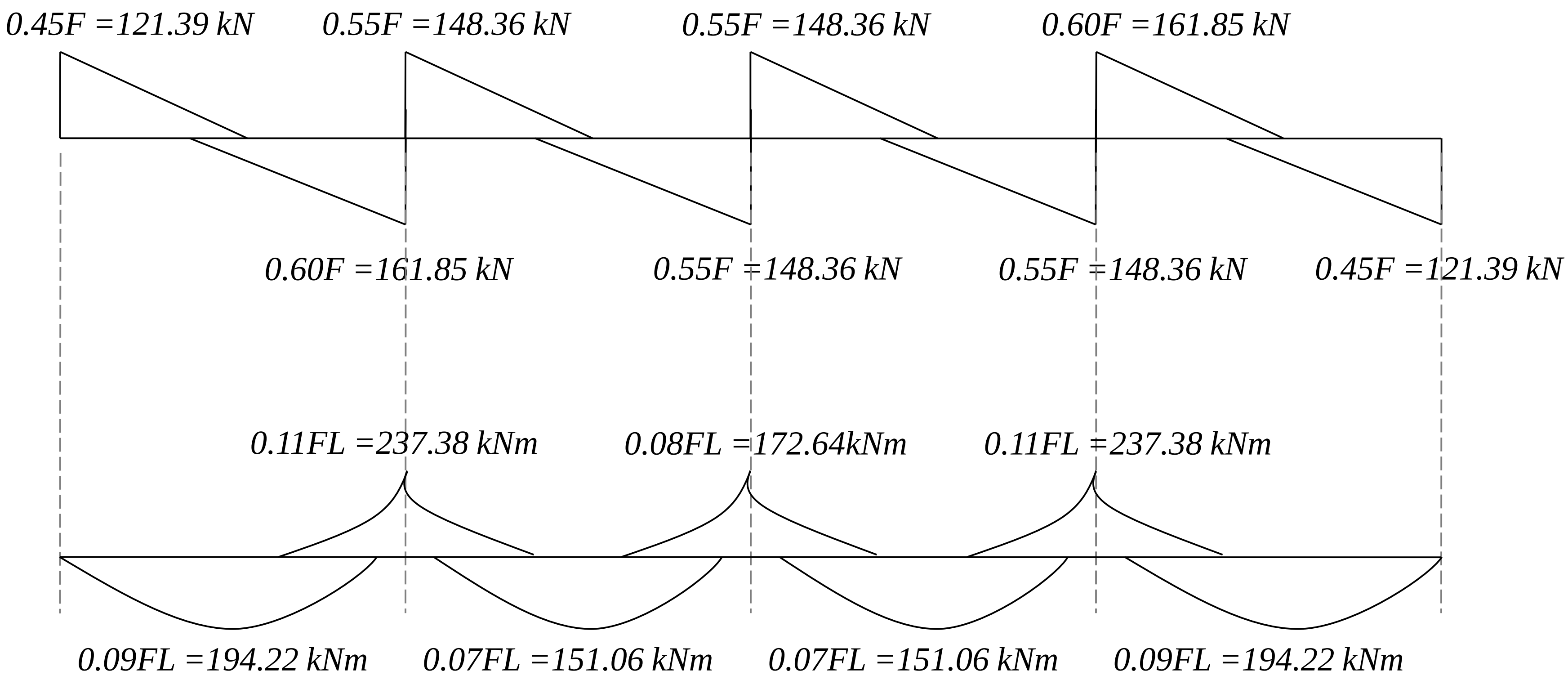


All unit in m and kN/m

$$F = 1.35G_k + 1.5Q_k$$
$$= 1.35(18.31) + 1.5(6.00) = 33.72\ kN/m \times 8\ m = 269.75\ kN$$

Example 2

Shear force and bending moment diagrams



Moment Redistribution

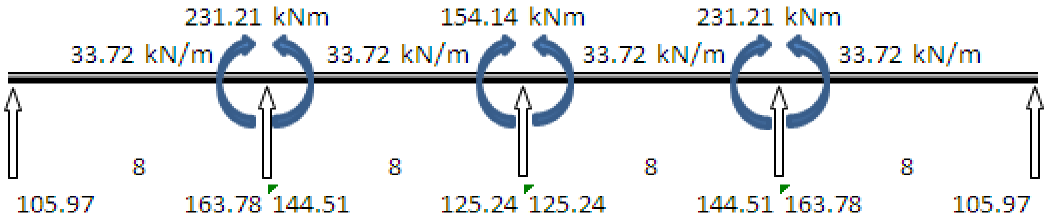
- Plastic behavior of RC at the ULS affects the distribution of moment in structure.
- To allow for this, the moment derived from an elastic analysis may be redistributed based on the assumption that plastic hinges have formed at the sections with the largest moment.
- From design point of view, some of elastic moment at support can be reduced, but this will increasing others to maintain the static equilibrium of the structure.
- The purpose or moment redistribution is to reduced the bending moment at congested zone especially at beam-column connection of continuous beam support.
- Therefore, the amount of reinforcement at congested zone also can be reduced then it will result the design and detailing process become much easier.

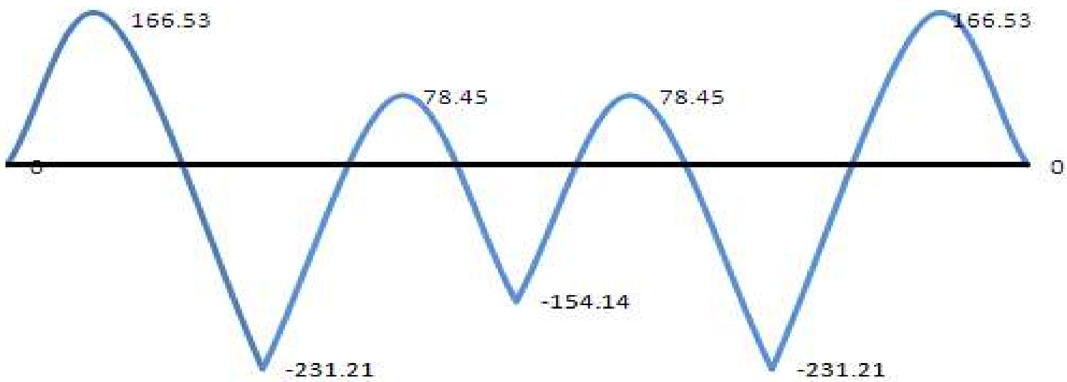
Moment Redistribution

- Section 5.5 EC2 permit the moment redistribution with the following requirement;
  - The resulting distribution remains in equilibrium with the load
  - The continuous beam are predominantly subject to flexural
  - The ratio of adjacent span should be in the range of 0.5 to 2
- There are other restrictions on the amount of moment redistribution in order to ensure ductility of the beam such as grade of reinforcing steel and area of tensile reinforcement and hence the depth of neutral axis.
  - Class A reinforcement; redistribution should ≤ 20%
  - Class B and C reinforcement; redistribution should ≤ 30%

Example 3

- For the moments obtained from Moment Distribution Method, redistribute 20% of moment at supports.





Example 3

Redistribute the moment at support

Original moment at support B & D = 231.21 kNm

Reduced moment (20%) = 0.8x231.21 = 184.97 kNm

Original moment at support C = 154.14 kNm

Reduced moment (20%) = 0.8x154.14 = 123.31 kNm

Recalculate the shear force using equilibrium principles.

Example 3

Span A - B

$\Sigma M_B = 0$

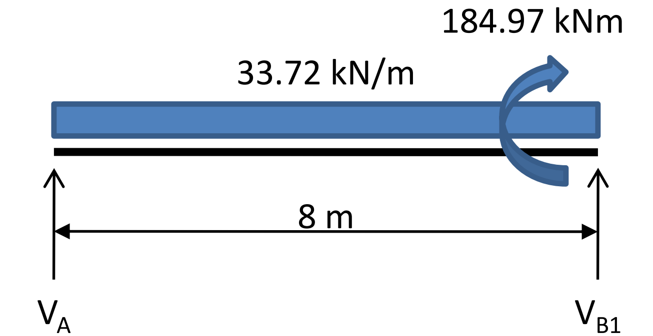
$V_A(8) – 33.72(8)^2/2 + 184.97 = 0$

$V_A = 894.07 / 8 = 111.76$ kN

$\Sigma F_y = 0$

$111.76 + V_{B1} – 33.72(8) = 0$

$V_{B1} = 158.0$ kN



Span B - C

$\Sigma M_C = 0$

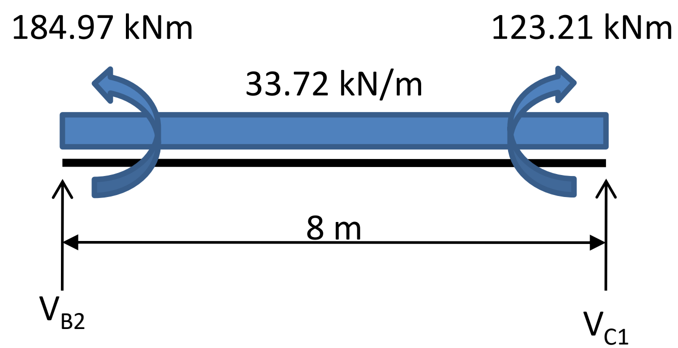
$V_{B2}(8) – 33.72(8)^2/2 + 123.21 - 184.97 = 0$

$V_{B2} = 1140.8 / 8 = 142.60$ kN

$\Sigma F_y = 0$

$142.60 + V_{C1} – 33.72(8) = 0$

$V_{C1} = 127.16$ kN

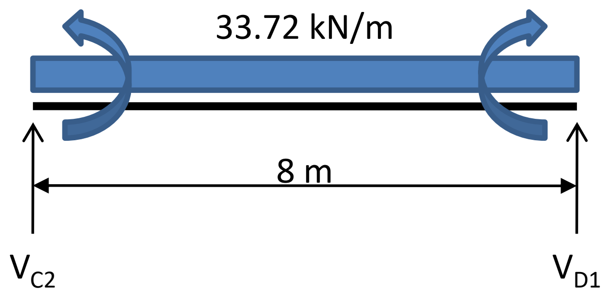


Example 3

Span C - D

$\Sigma M_D = 0$
$V_{C2}(8) – 33.72(8)^2/2 - 123.21 + 184.97 = 0$
$V_{C2} = 1017.28 / 8 = 127.16$ kN

$\Sigma F_y = 0$
$127.16 + V_{D1} – 33.72(8) = 0$
$V_{D1} = 142.60$ kN



Span D - E

$\Sigma M_E = 0$
$V_{D2}(8) – 33.72(8)^2/2 - 184.97 = 0$
$V_{D2} = 1264.01 / 8 = 158.0$ kN