Semester 1 2016/2017

Analysis of Section

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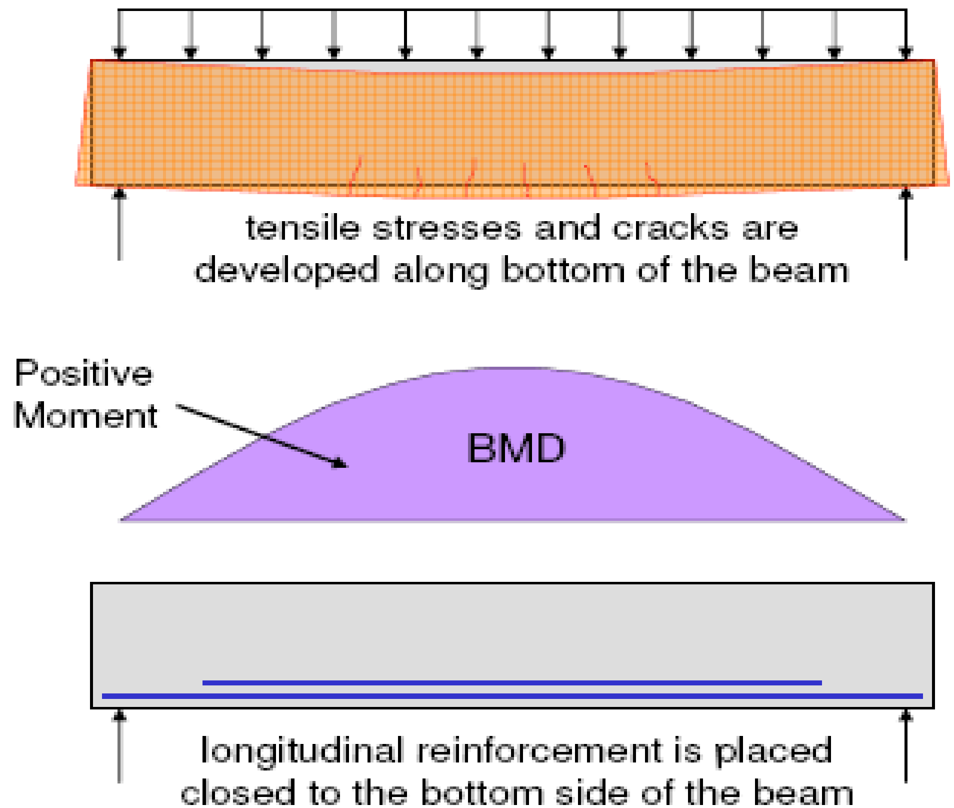
Introduction

- Behaviour of beam in bending: consider a simply supported beam subjected to gradually increasing load. The load causes the beam to bend and exert a bending moment.

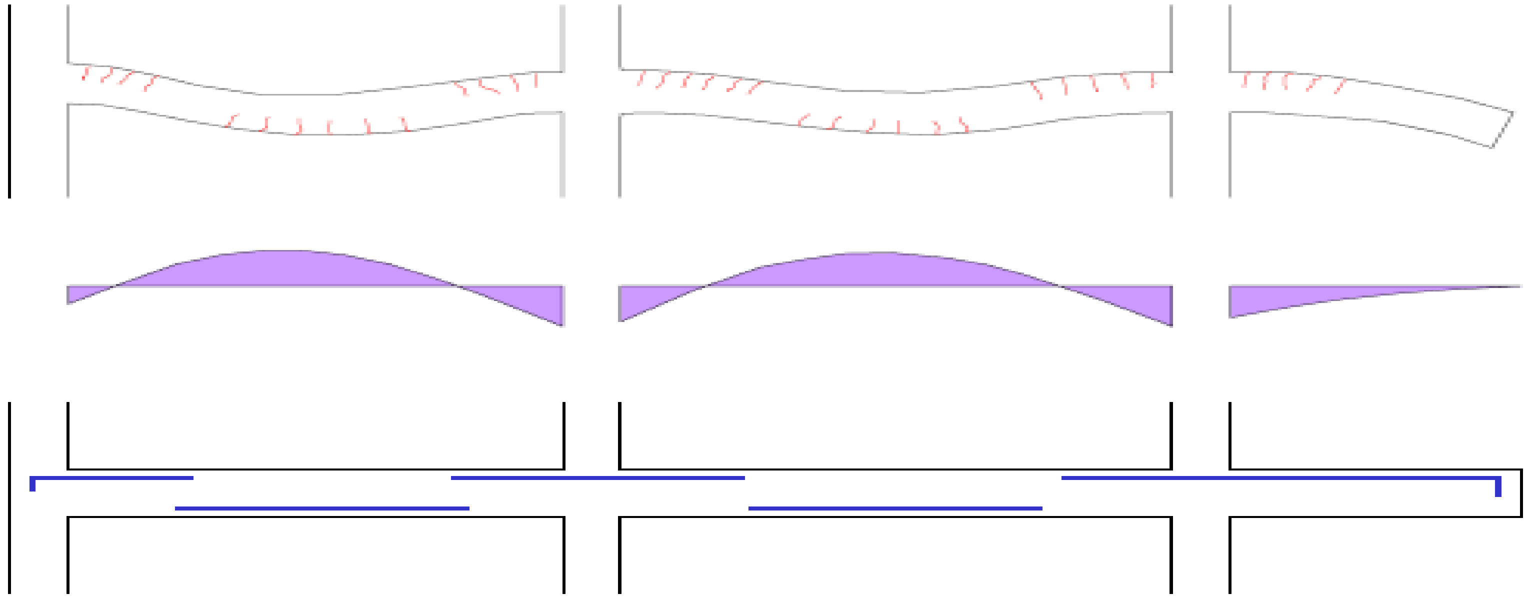
- The top surface of the beam is seen to shorten under compression, and the bottom surface lengthens under tension.

- As the concrete cannot solely resist tension (low f_t), steel reinforcement is introduces at the bottom surface to resist tension.



Introduction

- The loads also cause the continuous beam to bend downward between the support and upward bending over the support.
- This will produce tensile zone at both span and support. As the concrete cannot resist flexural tension, steel reinforcement would be introduced.



Basic Assumption

- In the design of reinforced concrete beam the following **assumptions** are made based on En 1991: Cl. 6.1.(2):

1) Plane section through the beam before bending remain plane after bending.

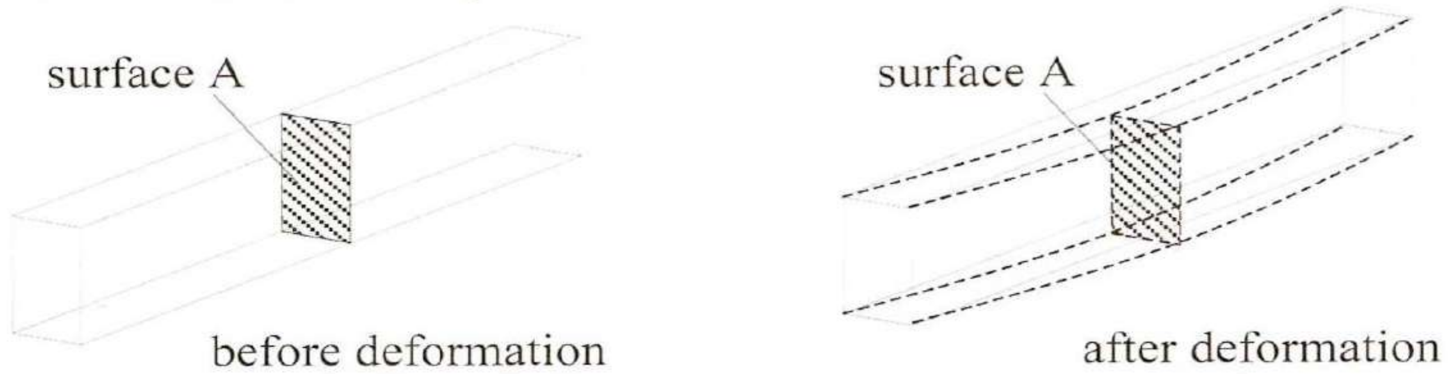
2) The strain in bonded reinforcement, whether in tension or compression is the same as that in the surrounding concrete.

3) The tensile stress in the concrete is ignored.

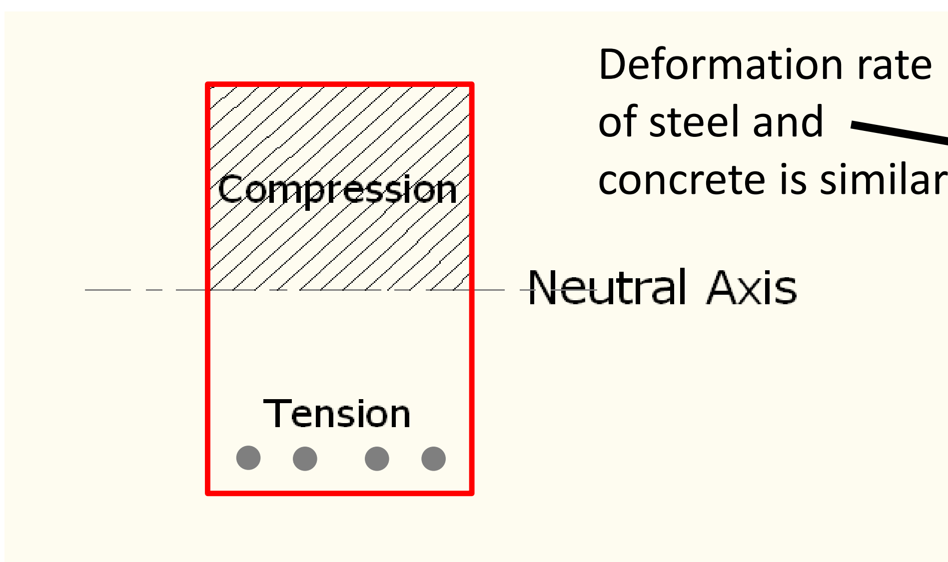
4) The stresses in the concrete and reinforcement can be derived from the strain by using stress-strain curve for concrete and steel.

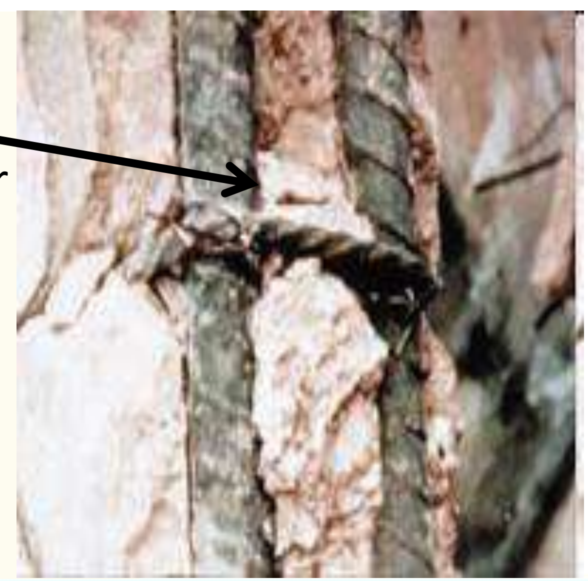
Basic Assumption

Plane sections remain plane



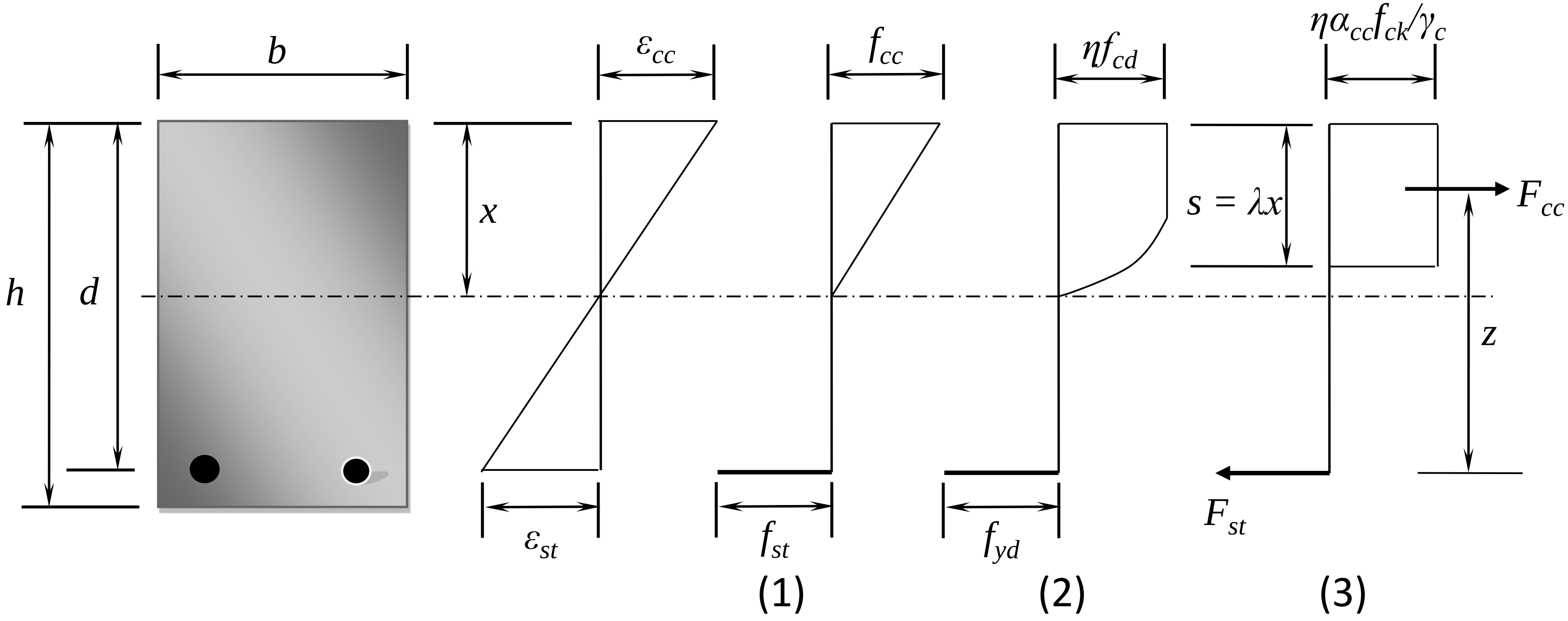
The surface of any cross-section does **not** distort out-of-plane during deformation.





Distribution of Stress-Strain

- The cross section of a RC beam subjected to bending and the resultant strain and stress distribution in the concrete.



For $f_{ck} < 50$ N/mm^2:

$\eta = 1$ (defining the effective strength), $\varepsilon_c = 0.0035$, $\alpha_{cc} = 0.85$,

$\lambda = 0.8$, $\gamma_c = 1.5$, $f_{cd} = 1.0 \times 0.85 \times f_{ck} / 1.5 = 0.567 f_{ck}$

Distribution of Stress-Strain

- Due to the tensile strength of concrete is very low, all the tensile stresses at the bottom fibre are taken by reinforcement.
- Stress distribution in the concrete:

1. The triangular stress distribution applies when the stress are very nearly proportional to the strain, which generally occurs at the loading levels encountered under working load conditions and is, therefore, used at the serviceability limit state.
2. The rectangular-parabolic stress block represents the distribution at failure when the compressive strain are within the plastic range, and it is associated with the design for ultimate limit state.
3. The equivalent rectangular stress block is a simplified alternative to the rectangular-parabolic distribution.

Distribution of Stress-Strain

- The distribution of strains across the beam cross section is linear. That is, the normal strain at any points in a beam section is proportional to its distance from the neutral axis.
- The steel strain in tension ε_{st} can be determined based on:

$$\frac{\varepsilon_{st}}{(d-x)} = \frac{\varepsilon_{cc}}{x} \Rightarrow \varepsilon_{st} = \varepsilon_{cc}\left(\frac{d-x}{x}\right) \quad \Rightarrow \quad x = \frac{d}{1+\left(\frac{\varepsilon_{st}}{\varepsilon_{cc}}\right)}$$

- Since, $\varepsilon_{cc} = 0.0035$ for class ≤ C50/60, steel with f_{yk} = 500 N/mm^2 and the yield strain is ε_{st} = 0.00217, by substituting ε_{cc} and ε_{st} ,thus $x=0.167d$.
- To ensure yielding of the tension steel at limit state the depth of neutral axis, x should be less than or equal to $0.617d$.

Failure of RC Beam

- As applied moment on the beam section increased beyond the linear elastic stage, the concrete strains and stresses enter the nonlinear stage.
- The behavior of the beam in the nonlinear stage depends on the amount of reinforcement provided.
- The reinforcing steel can sustain very high tensile strain however, the concrete can accommodate compressive strain much lower compare to it.
- So, the final collapse of a normal beam at ultimate limit state is cause by the crushing of concrete in compression, regardless of whether the tension steel has yield or not.
- Failure on RC beam may occur in the form of flexural, shear crack or dowel.

Failure of RC Beam

- Depending on the amount of reinforcing steel provided, flexural failure may occur in three ways:

1) **Balanced**: Concrete crushed and steel yields simultaneously at the ultimate limit state. The compressive strain of concrete reaches the ultimate strains ε_{cu} and the tensile strain of steel reaches the yield strain ε_{y} simultaneously. The depth of neutral axis, ***x = 0.617d***.

2) **Under-reinforced**: Steel reinforcement yields before concrete crushes. The area of tension steel provided is less than balance section. The depth of neutral axis, ***x < 0.617d***. The failure is gradual, giving ample prior warning of the impending collapse. This mode if failure is preferred in design practice.

Failure of RC Beam

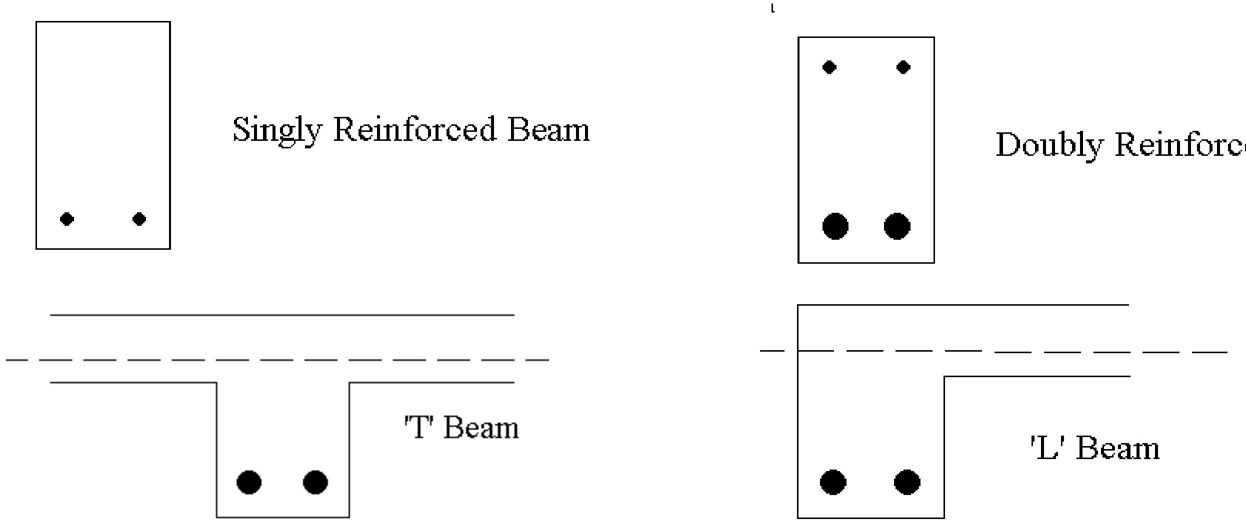
- Flexural failure (cont…):

  3) Over-reinforced: Concrete fails in compression before steel yields. The area of steel provided is more than area provided in balance section. The depth of neutral axis, ***x > 0.617d***. The failure is sudden (without any sign of warning) and brittle. Over-reinforced are not permitted.

- For a singly reinforced beam EC2 limits the depth to the neutral axis, ***x* to 0.45*d* (*x* ≤ 0.45*d*)** for concrete class ≤ *C50/60* to ensure that the design is for the under-reinforced case where failure is gradual.

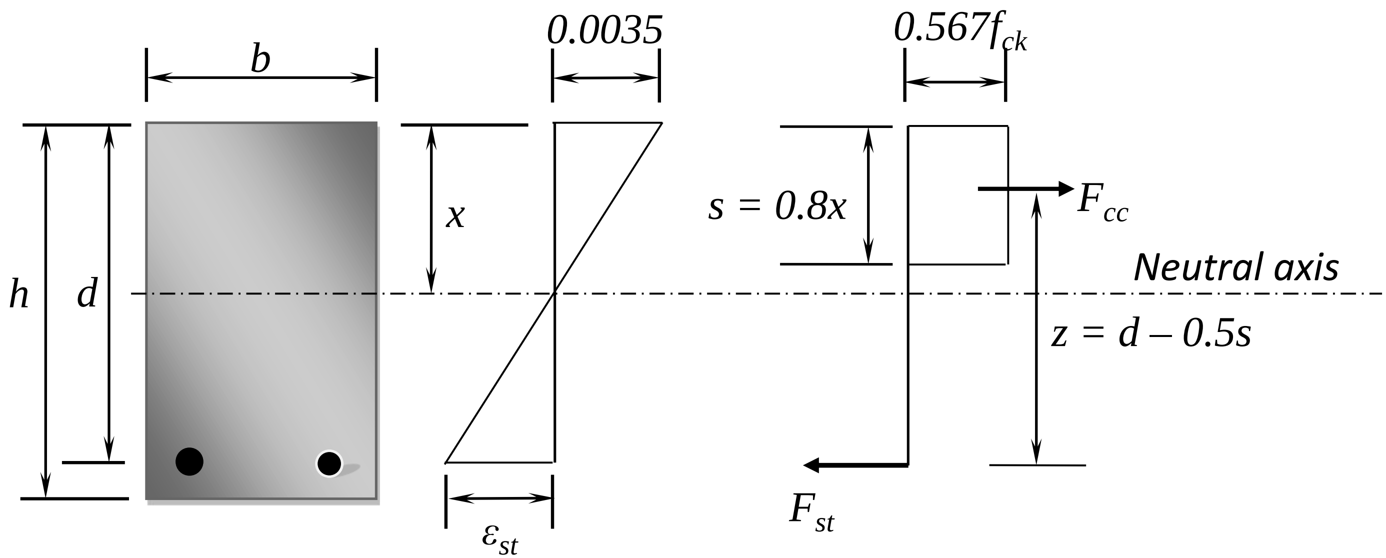
Analysis of Section

- Section 6.1 EN 1992-1-1, deal with the analysis and design of section for the ultimate limit state design consideration of structural elements subjected to bending.
- The two common types of reinforced concrete beam section are:
  1) Rectangular section : Singly and doubly reinforced
  2) Flanged section : Singly and doubly reinforced



Singly Reinforcement

- Beam cross section, strains and stresses distribution at ULS of singly reinforced rectangular beam:



Notation:

h = Overall depth

b = Width of section

A_s = Area of tension reinforcement

f_{ck} = Characteristic strength of concrete

f_{yk} = Characteristic strength of reinforcement

d = Effective depth

s = Depth of stress block

x = Neutral axis depth

z = Lever arm

Singly Reinforcement

Tension force of steel, F_{st}

F_{st} = Stress x Area = $0.87f_{yk} \times A_s$

Compression force of concrete, F_{cc}

F_{cc} = Stress x Area = $0.567f_{ck}\left(b \times 0.8x\right) = 0.454f_{ck}bx$

For equilibrium, total force in the section should be zero

$F_{st} = F_{cc}$ ⟶ $0.87f_{yk} \times A_s = 0.454f_{ck}bx$

$$x = \frac{0.87f_{yk}A_s}{0.454f_{ck}b}$$

Singly Reinforcement

Moment resistance with respect to the steel

$$M = F_{cc} \times z = (0.454 f_{ck} bx)(d - 0.4x)$$

$$M = \left(\frac{0.454x}{d}\right)\left(\frac{d - 0.4x}{d}\right)(f_{ck} bd^2)$$

Lets $\left(\frac{0.454x}{d}\right)\left(\frac{d - 0.4x}{d}\right) = K$ → $M = K f_{ck} bd^2$

Moment resistance with respect to the concrete

$$M = F_{st} \times z = (0.87 f_{yk} A_s)(d - 0.4x)$$

$$A_s = \frac{M}{0.87 f_{yk} (d - 0.4x)}$$

Required area of tension reinforcement

Singly Reinforcement

- To ensure that the section designed is under-reinforced it is necessary to place a limit on the maximum depth of the neutral axis (x). EC2 suggests ***$x \leq 0.45d$***.
- Then ultimate moment resistance of singly reinforced section or M_{bal} can be obtained by:

$$M_{bal} = (0.454 f_{ck} bx)(d - 0.4x)$$

$$M_{bal} = [0.454 f_{ck} b(0.45d)][d - 0.4(0.45d)]$$

$$M_{bal} = (0.2043 f_{ck} bd)(0.82d)$$

$$M_{bal} = 0.167 f_{ck} bd^2$$

$$M_{bal} = K_{bal} f_{ck} bd^2$$

Singly Reinforcement

- Therefore:

$$M = Kf_{ck}bd^2$$

$$M_{bal} = K_{bal}f_{ck}bd^2$$

where $K_{bal} = 0.167$

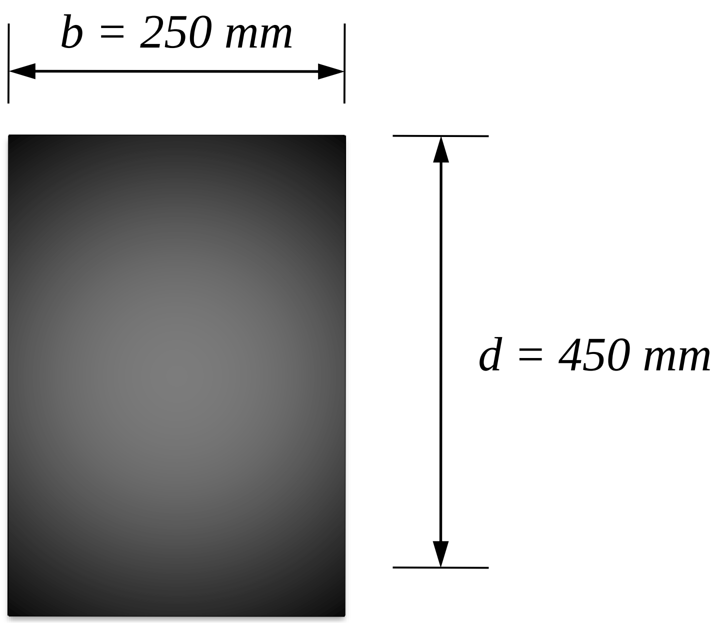
- If

$M \leq M_{bal}$ or $K \leq K_{bal}$ → Singly reinforced rectangular beam (tension reinforcement only)

$M > M_{bal}$ or $K > K_{bal}$ → Doubly reinforced rectangular beam (requires compression reinforcement)

Example 1

The cross section of rectangular beam is shown in the figure below. Using the stress block diagram and the following data, determine the area and the number of required reinforcement.



Design moment,
M_{Ed} = 200 kN.m

Chac. strength of concrete
f_{ck} = 25 N/mm^2

Chac. strength of steel
f_{yk} = 500 N/mm^2

Example 1

- Ultimate moment resistance of section:

$$M_{bal} = 0.167 f_{ck} b d^2$$

$$= 0.167(25)(250)(450^2)$$

$$= 211.36 kNm > M = 200 kNm$$

Singly reinforcement

- Neutral axis depth, x

$$M = \left(0.454 f_{ck} b x\right)\left(d - 0.4x\right)$$

$$200 \times 10^6 = 0.454(25)(250)(x)(450 - 0.4x)$$

$$x^2 - 1125x + 176211.45 = 0$$

$$x = 188mm \text{ or } 937mm$$

Example 1

- Use x = 188mm
- Check $\frac{x}{d} = \frac{188}{450} = 0.42 < 0.45$

< 0.617 under reinforcement

- Lever arm, $z = (d - 0.4x)$

$$= 450 - (0.4 \times 188) = 374.8mm$$

- Therefore, area of reinforcement

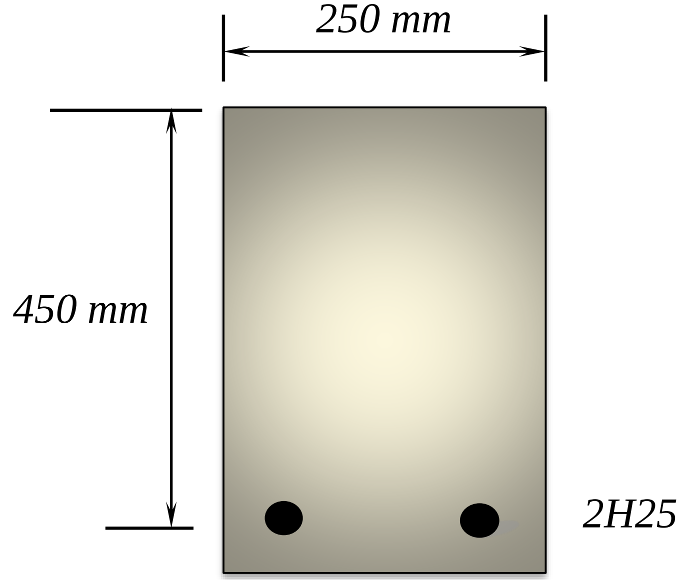
$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{200 \times 10^6}{0.87(500)(374.8)}$$

$$A_{s,req} = 1227mm^2$$

Provide $4H20\,(A_{s,prov} = 1257mm^2)$

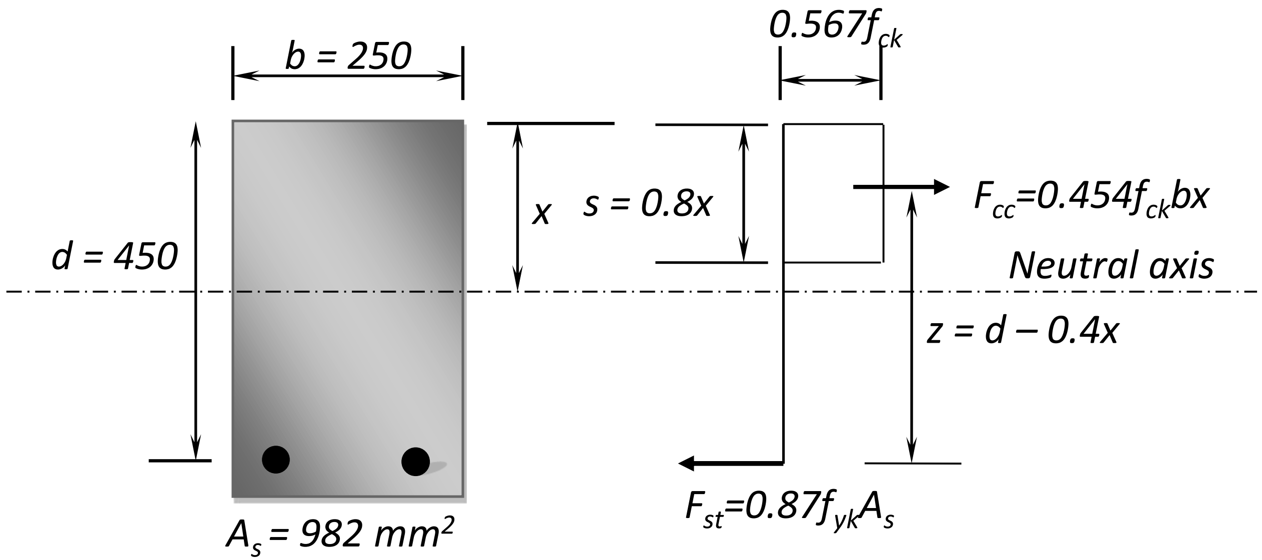
Example 2

Figure below shows the cross section of a singly reinforced beam. Determine the resistance moment for that cross section with the assistance of a stress block diagram. Given f_{ck}= 25 N/mm^2 and f_{yk} = 500 N/mm^2.



Example 2

- A stress block diagram is drawn with the important values and notations.



- For equilibrium

$$F_{cc} = F_{st}$$

$$0.454 f_{ck} bx = 0.87 f_{yk} A_s \quad \Rightarrow \quad x = \frac{0.87 f_{yk} A_s}{0.454 f_{ck} b}$$

Example 2

- Neutral axis depth, *x*

$$x = \frac{0.87(500)(982)}{0.454(25)(250)} = 151\,mm$$

- Check $\frac{x}{d} = \frac{151}{450} = 0.34 < 0.45$

- Moment resistance of section

$$M = F_{cc} \times z \quad \text{or} \quad M = F_{st} \times z$$

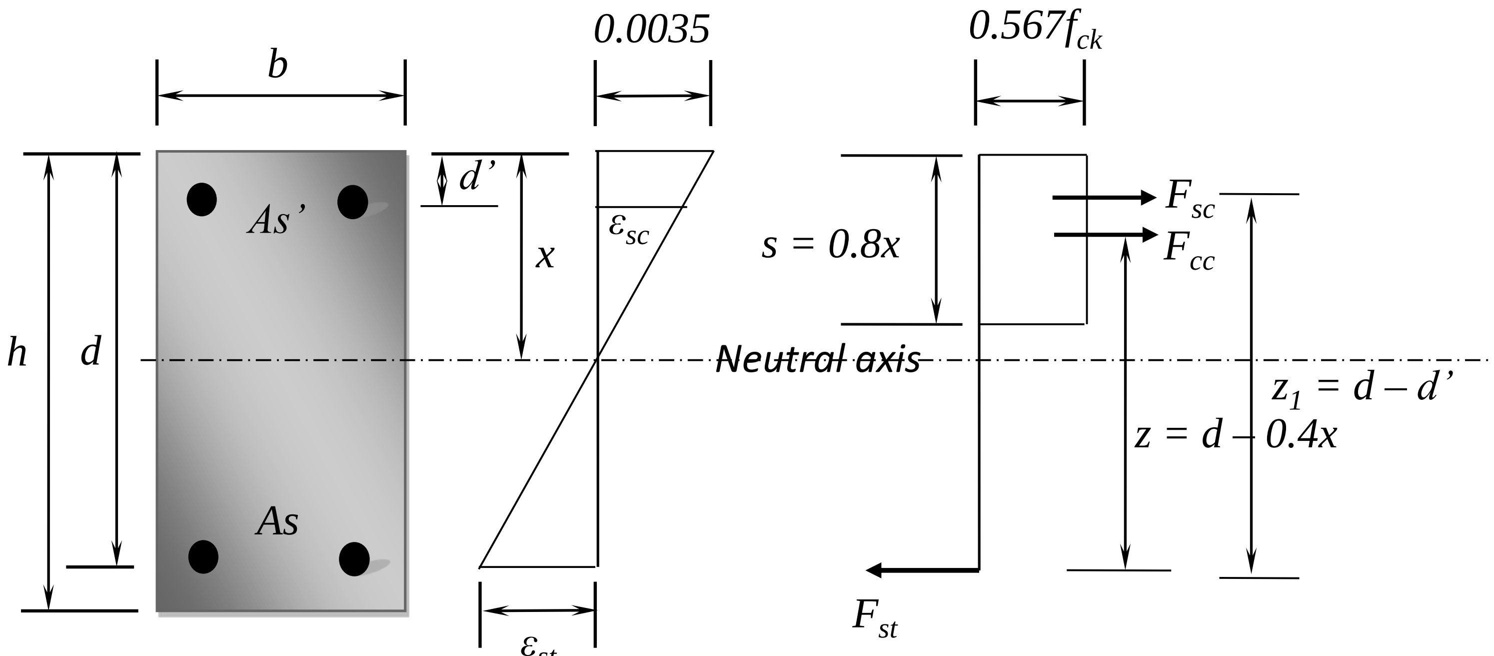
$$M = (0.454 f_{ck} b x)(d - 0.4x)$$

$$M = (0.454 \times 25 \times 250 \times 151)(450 - 0.4(151))$$

$$M = 167\ kNm$$

Doubly Reinforcement

- When the load applied increases gradually and it will reach a state that the compressive strength of concrete is not adequate to take additional compressive stress. Compression reinforcement is required to take the additional compressive stress. This section is named as doubly reinforced section and the stress-strain block diagram is:



Doubly Reinforcement

- Internal forces

$$F_{cc} = 0.454 f_{ck} bx$$

$$F_{st} = 0.87 f_{yk} A_s \quad \text{and} \quad F_{sc} = 0.87 f_{yk} A_s'$$

- Lever arm

$$z = d - 0.4x \quad \text{and} \quad z_1 = d - d'$$

- For equilibrium of internal force

$$F_{st} = F_{cc} + F_{sc}$$

$$0.87 f_{yk} A_s = 0.454 f_{ck} bx + 0.87 f_{yk} A_s'$$

Doubly Reinforcement

- Taking moment about the centroid of the tension steel

$$M = F_{cc}z + F_{sc}z_1$$

$$M = (0.454f_{ck}bx)(d - 0.4x) + (0.87f_{yk}A_s')(d - d')$$

- For design purpose, $x = 0.45d$

$$M = (0.454f_{ck}bx)[d - 0.4(0.45d)] + (0.87f_{yk}A_s')(d - d')$$

$$= 0.167f_{ck}bd^2 + (0.87f_{yk}A_s')(d - d')$$

$$= M_{bal} + (0.87f_{yk}A_s')(d - d')$$

- Therefore, the area of compression reinforcement

$$A_s' = \frac{(M - M_{bal})}{0.87f_{yk}(d - d')} \quad \text{or} \quad A_s' = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')}$$

Doubly Reinforcement

- In order to determine the area of tensile reinforcement, multiplied equilibrium internal force equation by *z*.
- Limiting $x = 0.45d$ and $z = d - 0.4(0.45d) = 0.82d$

$$0.87f_{yk}A_s z = 0.454f_{ck}bxz + 0.87f_{yk}A_s'z$$

$$0.87f_{yk}A_s z = 0.454f_{ck}b(0.45d)(0.82d) + 0.87f_{yk}A_s'z$$

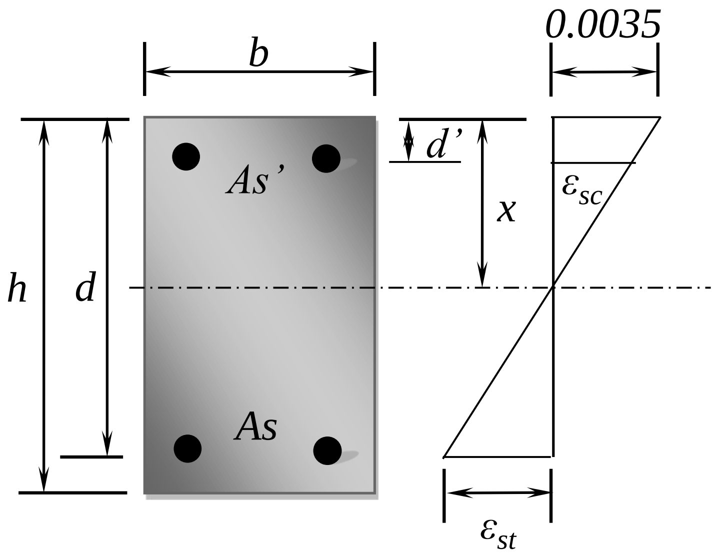
$$0.87f_{yk}A_s z = 0.167f_{ck}bd^2 + 0.87f_{yk}A_s'z$$

$$A_s = \frac{0.167f_{ck}bd^2}{0.87f_{yk}z} + A_s'$$

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A_s'$$

Doubly Reinforcement

- The derivation of design formula for doubly reinforced section assumed that the compression reinforcement reaches the design strength of $0.87f_{yk}$ at ultimate limit state.
- The strain diagram:



$$\frac{\varepsilon_{sc}}{(x - d')} = \frac{0.0035}{x}$$

$$\frac{(x - d')}{x} = \frac{\varepsilon_{sc}}{0.0035}$$

$$\frac{d'}{x} = 1 - \left(\frac{\varepsilon_{sc}}{0.0035} \right)$$

Doubly Reinforcement

- For the design strength $0.87f_{yk}$ to be reached, $\varepsilon_{sc} = 0.87f_{yk} / E_s$

$$\varepsilon_{sc} = \frac{0.87f_{yk}}{E_s} = \frac{0.87(500)}{200 \times 10^3} = 0.002175$$

$$\frac{d'}{x} = 1 - \left(\frac{0.002175}{0.0035}\right) = 0.38$$

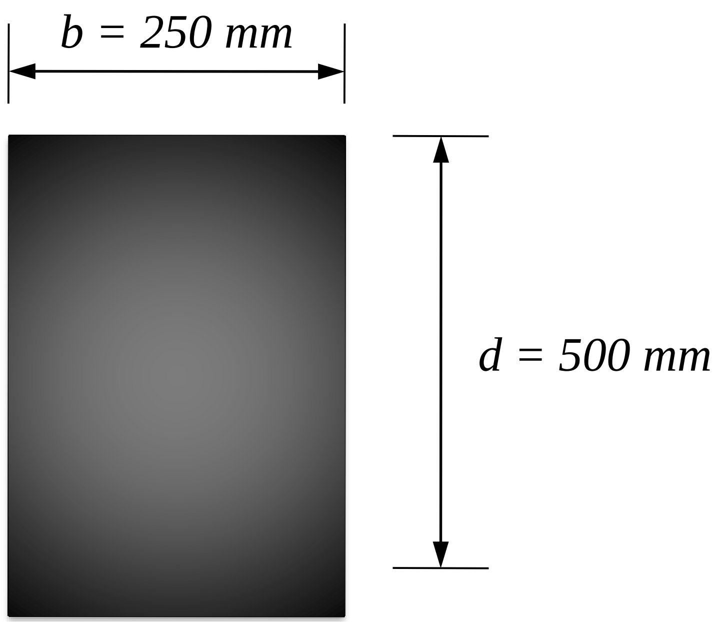
- Therefore, if $d'/x < 0.38$ the compression reinforcement can be assumed reach the design strength of $0.87f_{yk}$. If $d'/x > 0.38$, a reduced stress should be used.

$$f_{sc} = E_s.\varepsilon_{sc}$$

$$f_{sc} = 200 \times 10^3(0.0035)(1 - d'/x) = 700(1 - d'/x)$$

Example 3

The cross section of rectangular beam is shown in the igure below. Using the data given, determine the area and the number of required reinforcement.



Design moment,
M_{Ed} = 450 kN.m

Chac. strength of concrete
f_{ck} = 25 N/mm^2

Chac. strength of steel
f_{yk} = 500 N/mm^2

d' = 50mm

Example 3

- Ultimate moment resistance of section:

$$M_{bal} = 0.167 f_{ck} b d^2$$

$$= 0.167(25)(250)(500^2)(10^{-6})$$

$$= 260.94 kNm < M = 450 kNm$$

$$\therefore \text{Compression reinforcement is required}$$

- Area of compression reinforcement

$$A_s' = (M - M_{bal}) / 0.87 f_{yk} (d - d')$$

$$= (450 - 260.94) \times 10^6 / 0.87(500)(500 - 50)$$

$$= 966 mm^2$$

Example 3

- Check *d'/ x* ratio

$$x = 0.45d = 0.45(500) = 225mm$$

$$d'/x = 50/225 = 0.22 < 0.38$$

Compression steel achieved it design strength at $0.87f_{yk}$

- Area of tensile reinforcement

$$A_s = \left(\frac{M_{bal}}{0.87f_{yk}z}\right) + A_s' = \frac{260.94\times10^6}{0.87\times500\times(0.82\times500)} + 966$$

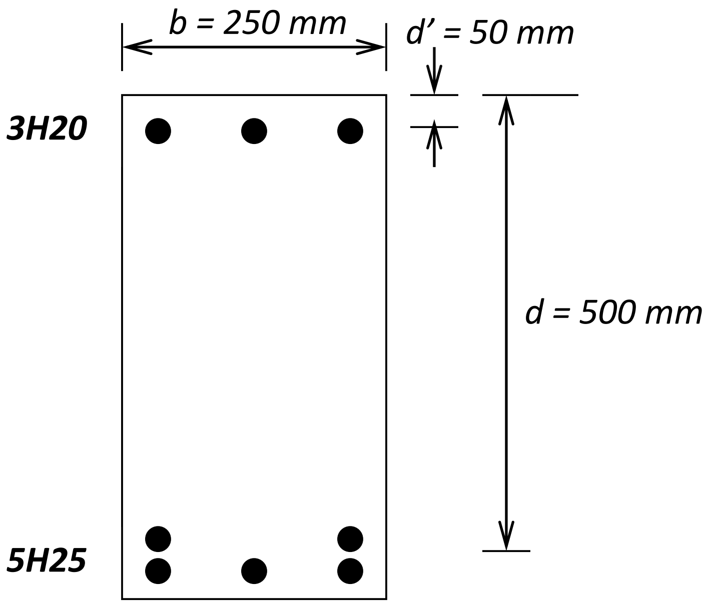
$$= 2429mm^2$$

Provide 2H25 ($A_{s'\,Prov.}$ = 982 mm²) Compression reinforcement

5H25 ($A_{s\,Prov.}$ = 2454 mm²) Tension reinforcement

Example 4

Calculate moment resistance of the doubly reinforced section shown in the figure below. Given the following data:



Chac. strength of concrete

f_{ck} = 30 N/mm^2

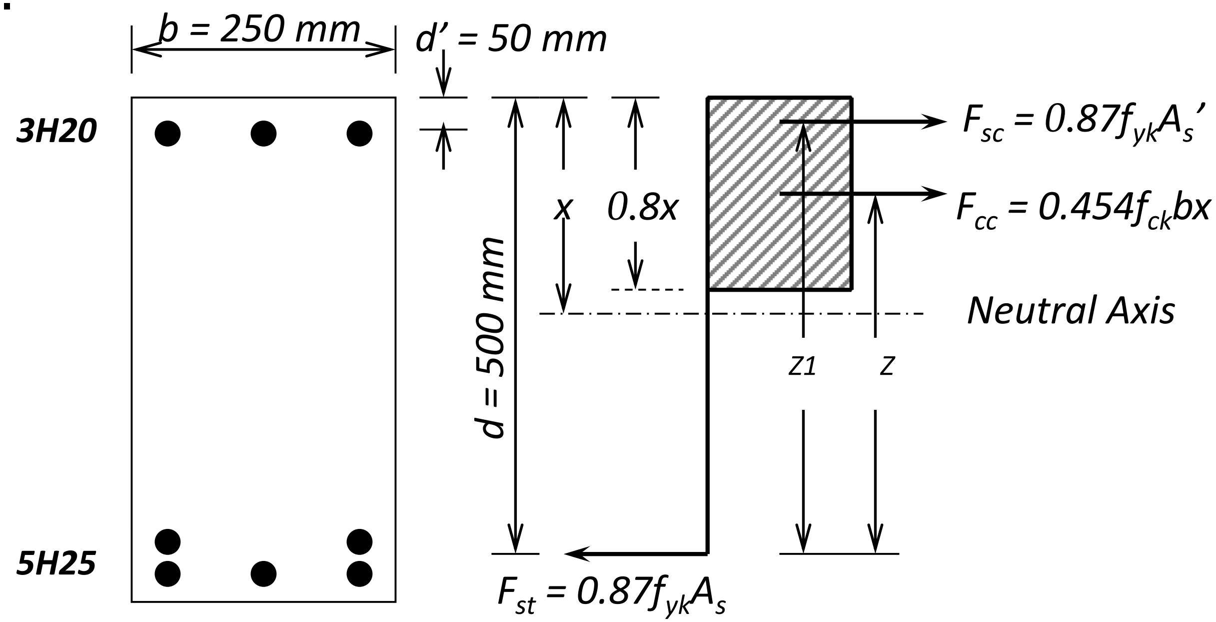
Chac. strength of steel

f_{yk} = 500 N/mm^2

d' = 50mm

Example 4

- A stress block diagram is drawn with the important values and notations.



- Reinforcement used 3H20 ($A_{s'}$ = *943 mm²*) and 5H25 (A_s = *2455 mm²*). *Neutral axis depth,*

$$x = \frac{0.87 f_{yk}(A_s - A_s')}{0.454 f_{ck} b} = \frac{0.87(500)(2455 - 943)}{0.454(30)(250)} = 193mm$$

Example 4

- Check the stress of steel

$$x / d = 193 / 500 = 0.39 < 0.45$$

$$d'/ x = 50 / 193 = 0.26 < 0.38$$

Steel achieved it design strength 0.87f_y

- Moment resistance of section

$$M = F_{sc}z_1 + F_{cc}z$$

$$= 0.87f_{yk}A_s'(d - d') + 0.454f_{ck}bx(d - 0.4x)$$

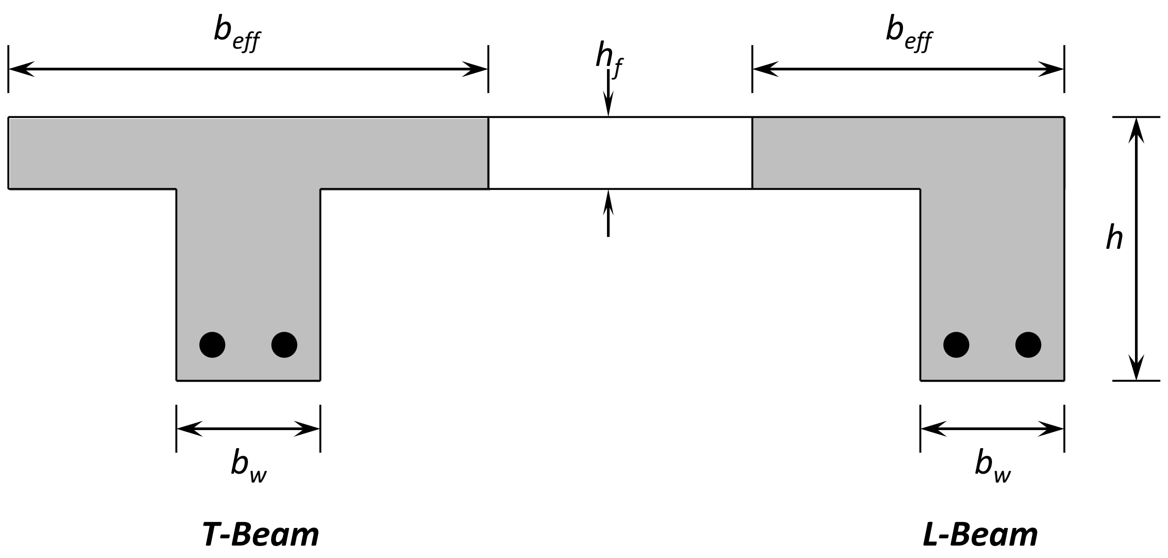
$$= 0.87(500)(943)(500 - 50) +$$

$$0.454(30)(250)(193)(500 - 0.4(193)) \times 10^{-6}$$

$$= 462kNm$$

Flange Beam

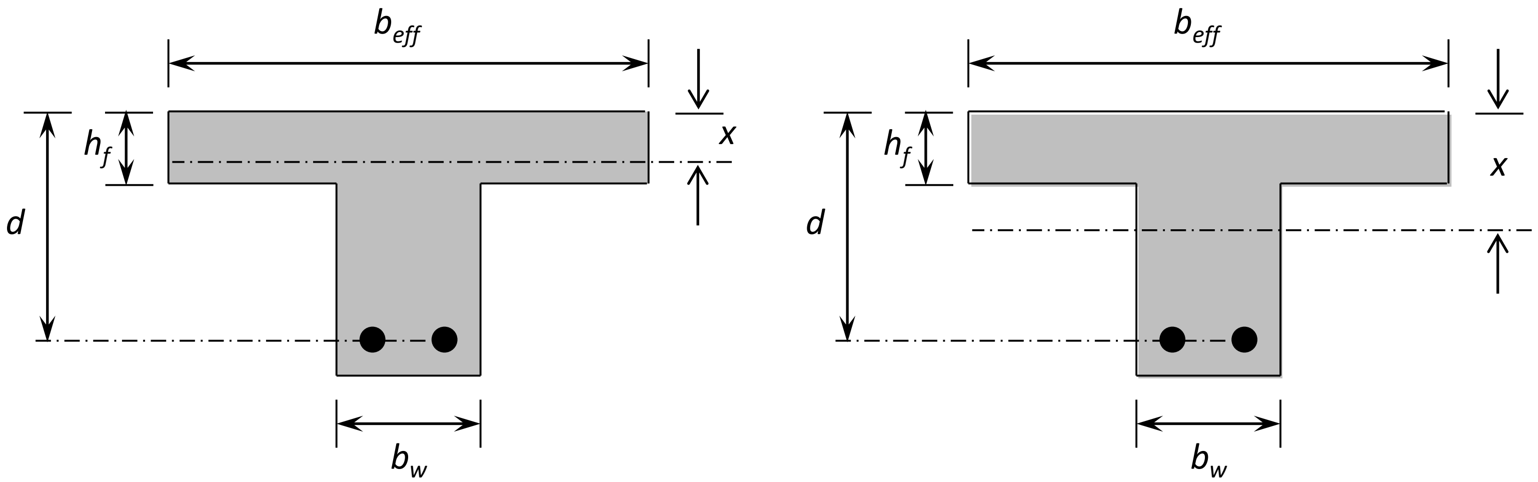
- Flanged beams occur when beams are cast integrally with and support a continuous floor slab.
- Part of the slab adjacent to the beam is counted as acting in compression to form T- and L-beam.
- The effective width of flange, b_{eff} is given in Sec. 5.3.2.1 of EC2 and should be based on the distance l_o.



where;
b_{eff} = effective flange width
b_w = breadth of the web of the beam.
h_f = thickness of the flange.

Flange Beam

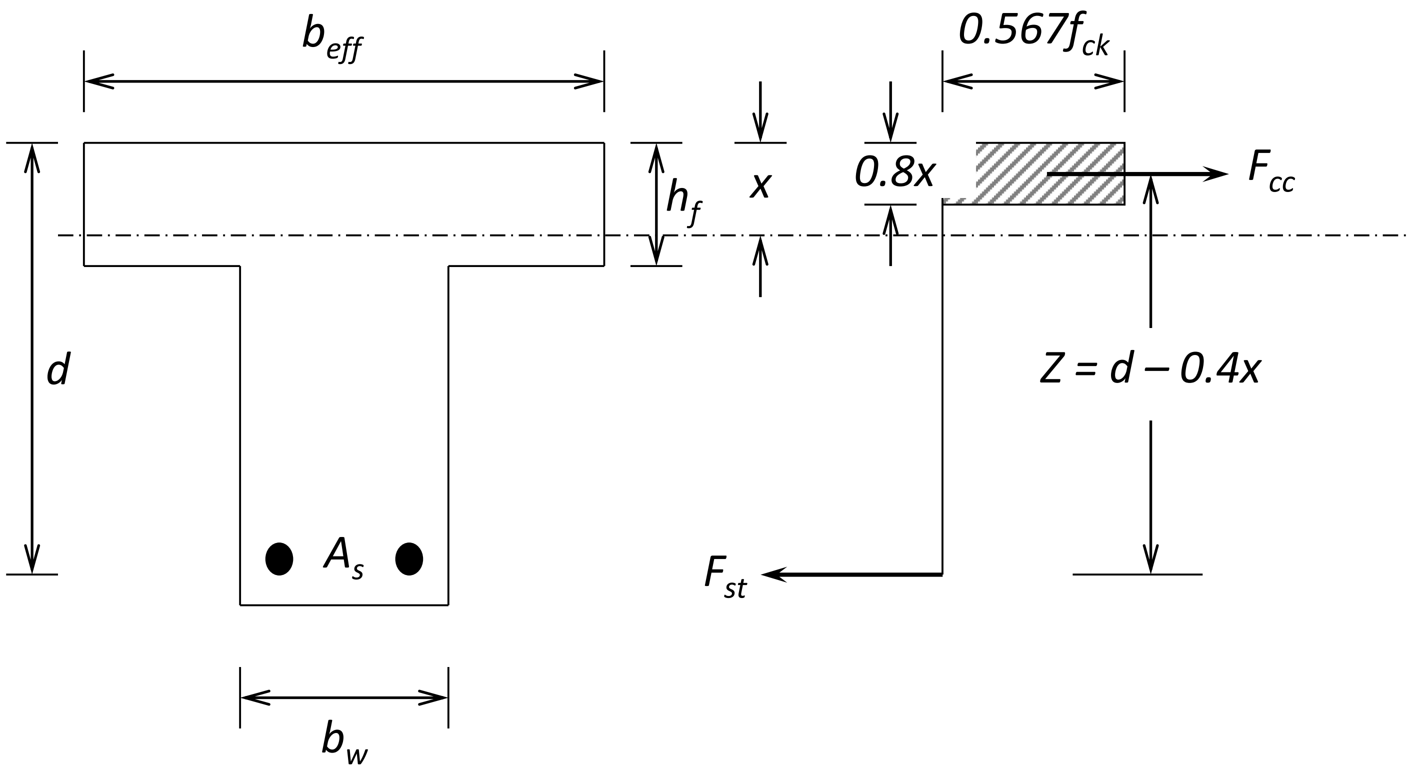
- The design procedure of flange beam depends on where the neutral axis lies.
- The neutral axis may lie in the flange or in the web.
- There are three cases that should be considered:
  - Neutral axis lies in flange *($M < M_f$)*
  - Neutral axis lies in web *($M > M_f$ but $< M_{bal}$)*
  - Neutral axis lies in web *($M > M_{bal}$)*



Flange Beam

- **Neutral axis lies in flange *($M < M_f$)***

  This condition occur when the depth of stress block *(0.8 x)* less then the thickness of flange, h_f.



Flange Beam

- Moment resistance of section, *M*

$$M = F_{cc} \times z$$

$$M = \left(0.567 f_{ck} b_{eff} 0.8x\right)\left(d - 0.4x\right)$$

- For this case, maximum depth of stress block, *0.8x* are equal to h_f

$$M = M_f = \left(0.567 f_{ck} b h_f\right)\left(d - h_f / 2\right)$$

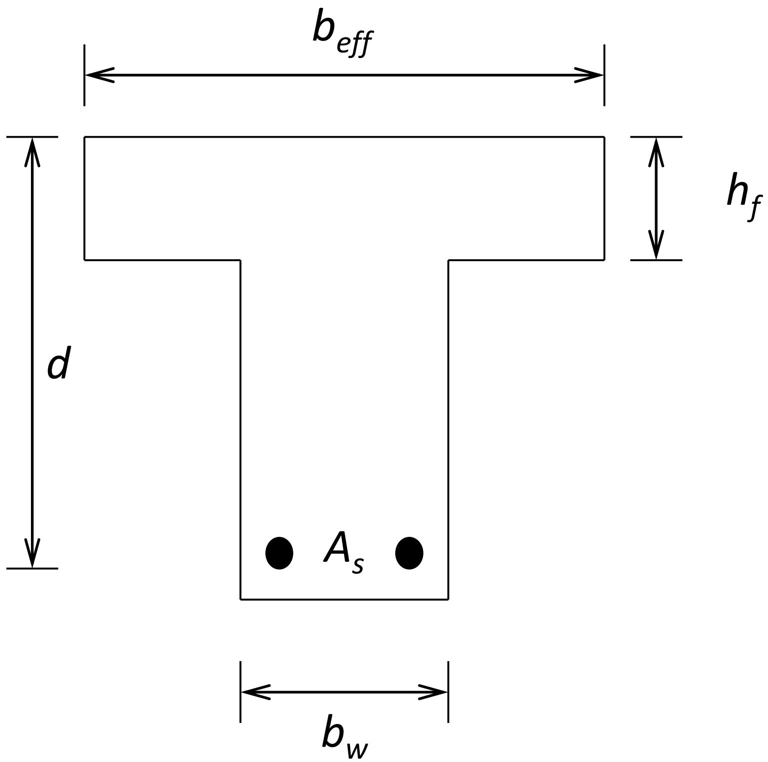
where, M_f = Ultimate moment resistance of flange.

- Therefore, if $M \leq M_f$ the neutral axis lies in flange and the design can be treated as rectangular singly reinforced beam.

$$A_s = \frac{M}{0.87 f_{yk} z} \quad \text{or} \quad A_s = \frac{M}{0.87 f_{yk}(d - 0.4x)}$$

Example 5

- A T-beam with dimension as shown in the figure below is subjected to design moment, $M = 250\ kNm$. Determine the required area and number of reinforcement if $f_{ck} = 30\ N/mm^2$ and $f_{yk} = 500\ N/mm^2$.



where;

b_{eff} =1450mm

b_w = 250mm

h_f = 100mm

d = 320mm

Example 5

- Moment resistance of flange, M_f

$$M_f = (0.567 f_{ck} b h_f)(d - h_f / 2)$$

$$M_f = (0.567 \times 30 \times 1450 \times 100)(320 - 100 / 2) \times 10^{-6}$$

$$M_f = 665.9 kNm > M = 250 kNm$$

Since $M < M_f$, Neutral axis lies in flange, and compression reinforcement is not required

- Neutral axis depth,

$$M = (0.454 f_{ck} b x)(d - 0.4x)$$

$$250 \times 10^6 = 0.454(30)(1450)(x)(320 - 0.4x)$$

$$x^2 - 800x + 31647.2 = 0$$

$$x = 758.3mm \quad @ \quad 41.74mm$$

Use x = 41.74mm

Example 5

- Checking

$$\frac{x}{d} = \frac{41.74}{320} = 0.13 < 0.45$$

- Lever arm,

$$z = d - 0.4x = 320 - 0.45(41.74) = 303.3\text{mm}$$

- Area of tension reinforcement

$$A_s = \frac{M}{0.87 f_{yk} z} = \frac{250 \times 10^6}{0.87(500)(303.3)}$$

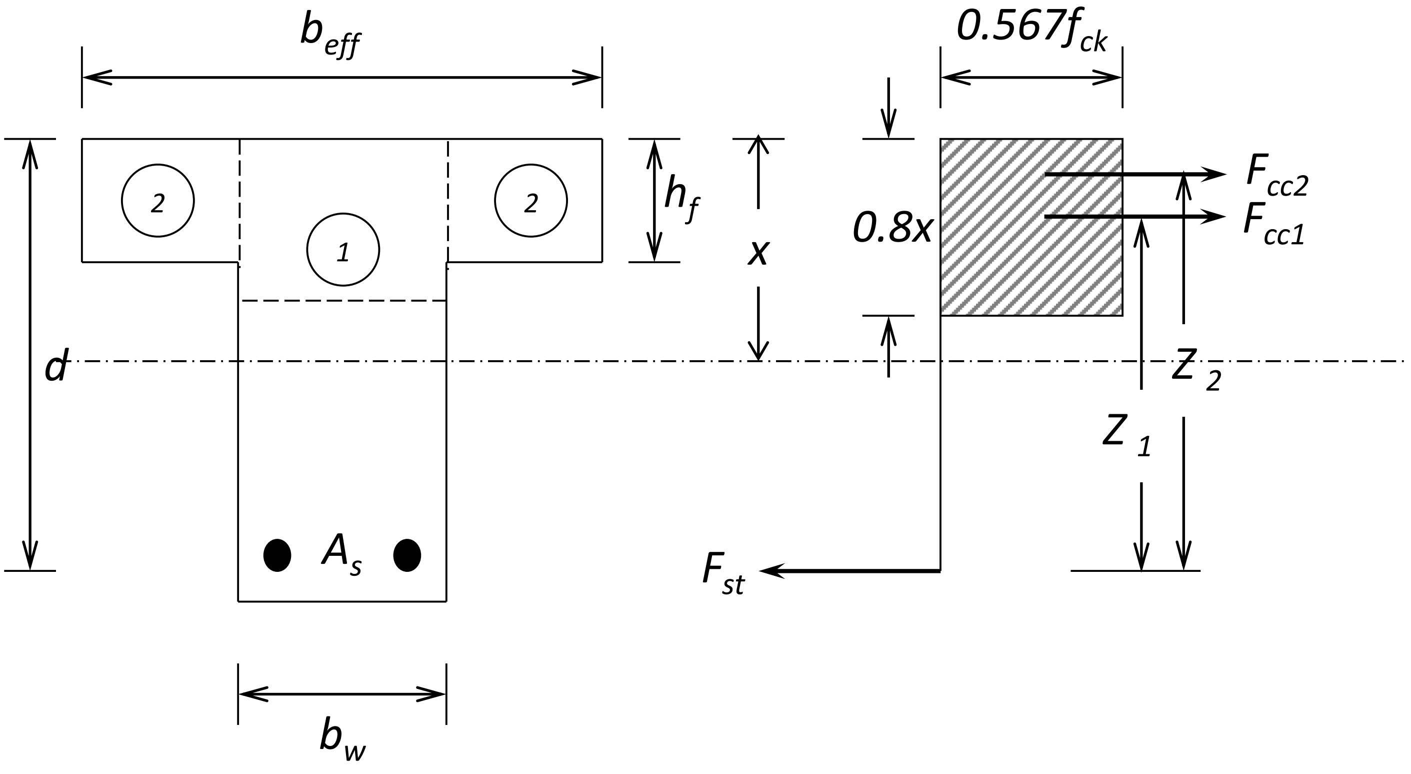
$$A_s = 1895 mm^2$$

Provide 4H25 (A_{sprov} = 1964 mm^2)

Flange Beam

- **Neutral axis lies in web** *($M_f < M < M_{bal}$)*

  If the applied moment M is greater than M_f the neutral axis lies in the web.



Flange Beam

- From the stress block, internal forces:

$$F_{cc1} = (0.567 f_{ck})(b_w 0.8x) = 0.454 f_{ck} b_w x$$

$$F_{cc2} = (0.567 f_{ck})(b_{eff} - b_w) h_f$$

$$F_{st} = 0.87 f_{yk} A_s$$

- Lever arm:

$$z_1 = d - 0.4x$$

$$z_2 = d - 0.5h_f$$

- Moment resistance:

$$M = F_{cc1} z_1 + F_{cc2} z_2$$

$$M = (0.454 f_{ck} b_w x)(d - 0.4x) + (0.567 f_{ck})(b_{eff} - b_w) h_f (d - 0.5h_f)$$

Flange Beam

- Ultimate moment resistance of section, *when x* = 0.45*d*:

$$M_{bal} = (0.454 f_{ck} b_w 0.45d)(d - 0.4(0.45d)) +$$
$$(0.567 f_{ck})(b_{eff} - b_w) h_f (d - 0.5h_f)$$

- Divide both side by $f_{ck} b_{eff} d^2$, then:

$$\frac{M_{bal}}{f_{ck} b_{eff} d^2} = 0.167 \frac{b_w}{b_{eff}} + 0.567 \frac{h_f}{d}\left(1 - \frac{b_w}{b_{eff}}\right)\left(1 - \frac{h_f}{2d}\right)$$

$$\frac{M_{bal}}{f_{ck} b_{eff} d^2} = \beta_f$$

Therefore, $M_{bal} = \beta_f f_{ck} b_{eff} d^2$

Flange Beam

- If applied moment $M < M_{bal}$, then compression reinforcement are not required.
- Area of tension reinforcement can be calculate as follows by taking moment at F_{cc2}.

$$M = F_{st} z_2 - F_{cc1}(z_2 - z_1)$$

$$M = 0.87 f_{yk} A_s (d - 0.5h_f) - 0.454 f_{ck} b_w x[(d - 0.5h_f) - (d - 0.4x)]$$

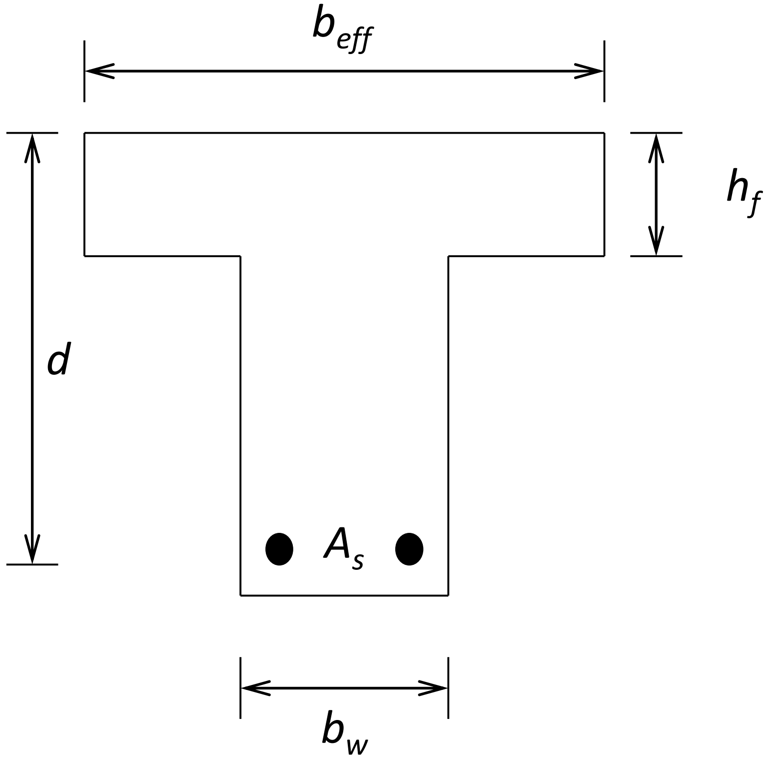
$$A_s = \frac{M + 0.454 f_{ck} b_w x[0.4x - 0.5h_f]}{0.87 f_{yk}(d - 0.5h_f)}$$

- Using; $x = 0.45d$

$$A_s = \frac{M + 0.1 f_{ck} b_w d[0.36d - h_f]}{0.87 f_{yk}(d - 0.5h_f)}$$

Example 6

- A T-beam with dimension as shown in the figure below is subjected to design moment, $M = 670\ kNm$. Determine the required area and number of reinforcement if $f_{ck} = 30\ N/mm^2$ and $f_{yk} = 500\ N/mm^2$.



where;

b_{eff} =1450mm

b_w = 250mm

h_f = 100mm

d = 320mm

Example 6

- Moment resistance of flange, M_f

$$M_f = (0.567 f_{ck} b h_f)(d - h_f / 2)$$

$$M_f = (0.567 \times 30 \times 1450 \times 100)(320 - 100/2) \times 10^{-6}$$

$$M_f = 665.9 kNm < M = 670 kNm$$

Since $M > M_f$, neutral axis lies in web

$$M_{bal} = \beta_f f_{ck} b_{eff} d^2$$

$$\beta_f = 0.167 \frac{250}{1450} + 0.567 \frac{100}{320}\left(1 - \frac{250}{1450}\right)\left(1 - \frac{100}{2(320)}\right)$$

$$\beta_f = 0.153$$

$$M_{bal} = 0.153(30)(1450)(320^2) \times 10^{-6}$$

Example 6

$$M_{bal} = 682kNm > M = 670kNm$$

Compression reinforcement is not required

- Area of tension reinforcement

$$A_s = \frac{M + 0.1 f_{ck} b_w d[0.36d - h_f]}{0.87 f_{yk}(d - 0.5h_f)}$$

$$A_s = \frac{670 \times 10^6 + 0.1(30)(250)(320)[0.36(320) - 100]}{0.87(500)(320 - 50)}$$

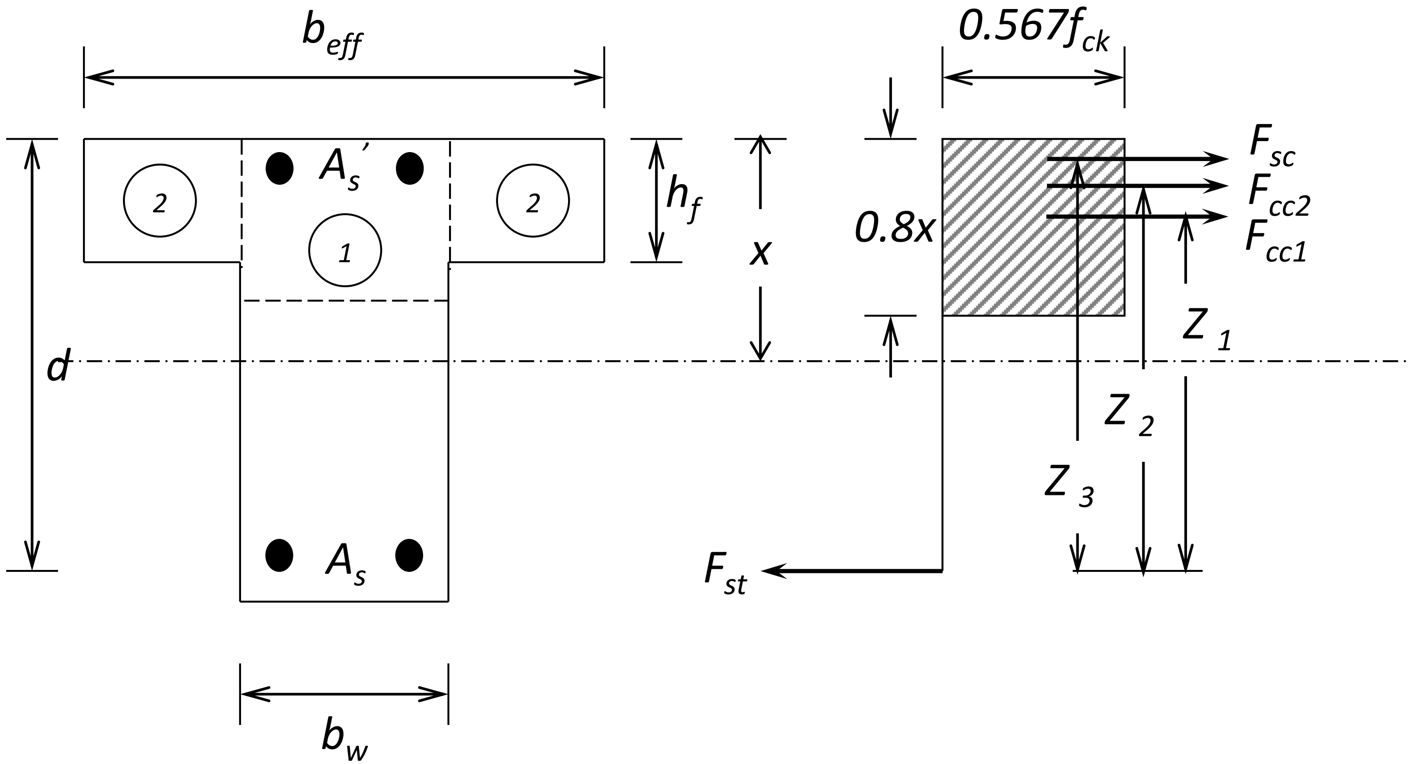
$$A_s = 5736mm^2$$

Provide 8H32 ($A_{s,pro}$v = 6433mm^2)

Flange Beam

- **Neutral axis lies in web** *($M > M_{bal}$)*

  If the applied moment M is greater than M_{bal} the neutral axis lies in the web and the compression reinforcement should be provided.



Flange Beam

- From the stress block, internal forces:

$$F_{cc1} = (0.567 f_{ck})(b_w 0.8x) = 0.454 f_{ck} b_w x$$

$$F_{cc2} = (0.567 f_{ck})(b_{eff} - b_w) h_f$$

$$F_{sc} = 0.87 f_{yk} A_s'$$

$$F_{st} = 0.87 f_{yk} A_s$$

- Lever arms:

$$z_1 = d - 0.4x \quad ; \quad z_2 = d - 0.5h_f \quad ; \quad z_3 = d - d'$$

- Moment resistance:

$$M = F_{cc1} z_1 + F_{cc2} z_2 + F_{sc} z_3$$

$$M = (0.454 f_{cu} b_w x)(d - 0.4x) +$$
$$(0.567 f_{cu})(b_{eff} - b_w) h_f (d - 0.5h_f) + 0.87 f_{yk} A_s'(d - d')$$

Flange Beam

- When $x = 0.45d$:

$$M = M_{bal} + 0.87 f_{yk} A_s'(d - d')$$

- Area of compression reinforcement:

$$A_s' = \frac{M - M_{bal}}{0.87 f_{yk}(d - d')}$$

- For equilibrium force

$$F_{st} = F_{cc1} + F_{cc2} + F_{sc}$$

$$0.87 f_{yk} A_s = 0.454 f_{ck} b_w (0.45d) + 0.567 f_{ck}(b_{eff} - b_w) + 0.87 f_{yk} A_s'$$

- Area of tension reinforcement

$$A_s = \frac{0.2 f_{ck} b_w d + 0.567 f_{ck} h_f (b_{eff} - b_w)}{0.87 f_{yk}} + A_s'$$