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Rectangular Beam Design

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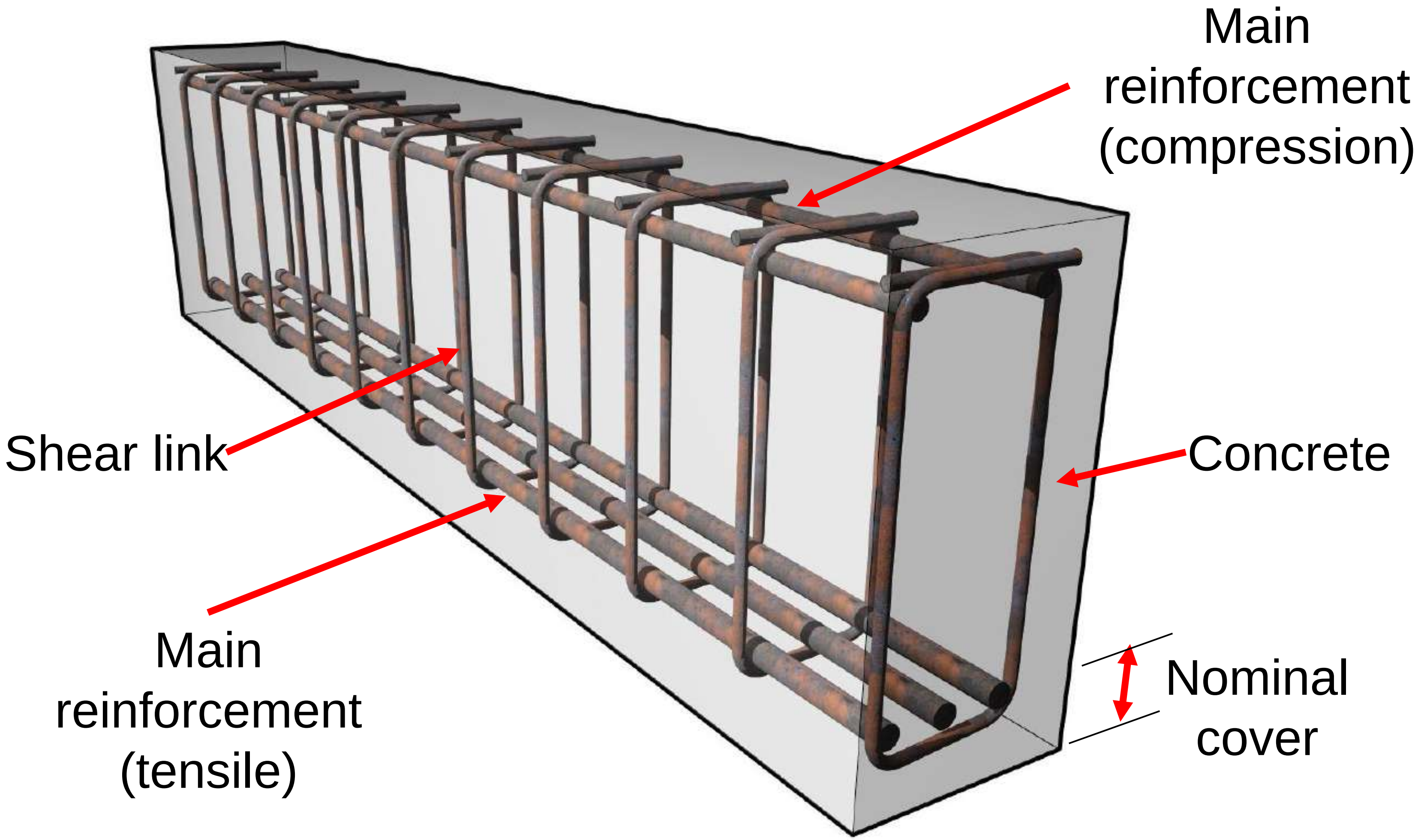
With Wisdom We Explore

Introduction

- The purposes of this chapter is to compile the design principles that have been previously discussed in order to form a complete design procedures of reinforced concrete beam.
- Basically, beam is the structural element which subjected to transverse load in the form of bending moment, shear force and torsion.
- Therefore, beam is designed to resist all that particular factors.
- Beam is also designed to fulfill the serviceability requirements in order to produce an adequate and safe design.
- The design principle of beam involves the design of main and secondary reinforcements, shear link, deflection control and cracking.

Introduction



Design Procedures

Step	Task	Standard
1	Determine design life	EN 1990:2002 Table 2.1
2	Determine beam size	EN 1992-1-1: Table 7.4N EN 1992-1-2: Table 5.5 & 5.6
3	Determine design actions on beam	EN 1991-1-1
4	Assess durability and characteristic strengths	EN 1992-1-1: Sec. 3 & 4
5	Determine nominal cover	EN 1991-1-1: Sec.4.4.1
6	Calculate moment and shear force	EN 1992-1-1: Sec.5
7	Design of flexural reinforcement	EN 1992-1-1: Sec.6.1, 9.2.1.1
8	Design of shear reinforcement	EN 1992-1-1: Sec.6.2
9	Check deflection	EN 1992-1-1: Sec.7.4
10	Check cracking	EN 1992-1-1: Sec.7.3
11	Detailing	EN 1992-1-1: Sec.8 & 9.2

Size of Beam

- From structural point of view, the selection of beam sizes is often dictated by deflection control criteria.
- Span to overall depth (*L*/*h*) ratios of 13 to 18 are generally found to be economical in the case of simply supported and continuous beam.
- The recommended ratio of width to overall depth (*b*/*h*) in rectangular beam section is in the range of 0.3 to 0.6.
- Beside of that, the beam sizes also control by an architectural detailing and fire resistance requirement.
- EC2 gives recommendation to determine the dimension of beam based on:

1) Minimum width, b_{min} based on fire resistance (Tables 5.5 and 5.6, BS EN 1992-1-2)
2) Effective depth, d_{eff} for deflection control (Table 7.4N)

Size of Beam

- Minimum width, b_{min} of beam (simply support) based on fire

Standard Fire Resistance	Minimum Dimension (mm)						
	Possible combinations of a and b_{min} where a is the average axis distance and b_{min} is the width of beam				Web Thickness b_w		
					Class WA	Class WB	Class WC
1	2	3	4	5	6	7	8
R 30	b_{min} = 80 a = 25	120 20	160 15*	200 15*	80	80	80
R 60	b_{min} = 120 a = 40	160 35	200 30	300 25	100	80	100
R 90	b_{min} = 150 a = 55	200 45	300 40	400 35	110	100	100
R 120	b_{min} = 200 a = 65	240 60	300 55	500 50	130	120	120
R 180	b_{min} = 240 a = 80	300 70	400 65	600 60	150	150	140
R 240	b_{min} = 280 a = 90	350 80	500 75	700 70	170	170	160

Size of Beam

- Minimum width, b_{min} of continuous beam based on fire

Standard Fire Resistance	Minimum Dimension (mm)						
	Possible combinations of a and b_{min} where a is the average axis distance and b_{min} is the width of beam				Web Thickness b_w		
					Class WA	Class WB	Class WC
1	2	3	4	5	6	7	8
R 30	b_{min} = 80 a = 15*	160 12*			80	80	80
R 60	b_{min} = 120 a = 25	200 12*			100	80	100
R 90	b_{min} = 150 a = 35	250 25			110	100	100
R 120	b_{min} = 200 a = 45	300 35	450 35	500 30	130	120	120
R 180	b_{min} = 240 a = 60	400 50	550 50	600 40	150	150	140
R 240	b_{min} = 280 a = 70	500 60	650 60	700 50	170	170	160

Size of Beam

- Effective depth, d_{eff} of beam based on deflection control

Structural System		K	Basic span-effective depth ratio	
			Concrete highly stressed, ρ=1.5%	Concrete lightly stressed, ρ=0.5%
1	Simply supported beam, one/two-way simply supported slab	1.0	14	20
2	End span of continuous beam or one-way continuous slab or two-way spanning slab continuous over one long side	1.3	18	26
3	Interior span of beam or one-way or two-way spanning slab	1.5	20	26
4	Slab supported on columns without beam (flat slab) based on longer span	1.2	17	24
5	Cantilever	0.4	6	8

Design For Flexural

- Design for flexural includes the determination of main reinforcement for tensile and compression, if required, and the limitation area of reinforcement.
- The derived formulae of flexural design is based on the rectangular stress block, Cl.3.1.7 and Cl.3.1.8.
- The area of tension reinforcement should not be taken as less than $A_{s,min}$ and should not exceed $A_{s,max}$, Cl.9.2.1.

$$A_{s,\min} < A_{s,prov} < A_{s,\max}$$

$$A_{s,\min} = 0.26\left(\frac{f_{ctm}}{f_{yk}}\right)b_t d \geq 0.0013 b_t d$$

$$A_{s,\max} = 0.04 A_c = 0.04 b_t h$$

Design For Flexural

- Design procedure for rectangular beam:

1) Calculate K,

$$K = \frac{M_{Ed}}{f_{ck} b d^2}$$

2) Calculate K_{bal}

$$K_{bal} = 0.363(\delta - 0.44) - 0.116(\delta - 0.44)^2$$

when moment redistribution is zero, $\delta = 1$, $K_{bal} = 0.167$

3) Comparison between K and K_{bal}

If $K \leq K_{bal}$, compression reinforcement is not required

$$z = d\left[0.5 + \sqrt{0.25 - \frac{K}{1.134}}\right] \leq 0.95d \; ; \; A_s = \frac{M_{Ed}}{0.87 f_{yk} z}$$

Design For Flexural

If $K > K_{bal}$, compression reinforcement is required, thus

$$z = d\left[0.5 + \sqrt{0.25 - \frac{K_{bal}}{1.134}}\right] \leq 0.95d$$

$$x = \frac{d - z}{0.4}$$

Checking, $\frac{d'}{x} \leq 0.38 \Rightarrow A_s' = \frac{(K - K_{bal})f_{ck}bd^2}{0.87f_{yk}(d - d')}$

$\frac{d'}{x} > 0.38 \Rightarrow A_s' = \frac{(K - K_{bal})f_{ck}bd^2}{f_{sc}(d - d')}$

$$A_s = \frac{K_{bal}f_{ck}bd^2}{0.87f_{yk}z} + A_s'\left(\frac{f_{sc}}{0.87f_{yk}}\right)$$

f_{sc} ?

Design For Flexural

- Design procedure for flange beam:
  1) Calculate M_f,

$$M_f = 0.567 f_{ck} b h_f (d - 0.5 h_f)$$

  2) If M ≤ Mf, neutral axis in the flange and hence provide tensile reinforcement only

$$K = \frac{M_{Ed}}{b d^2 f_{ck}}$$

$$z = d\left[0.5 + \sqrt{0.25 - \frac{K}{1.134}}\right] \leq 0.95d$$

$$A_{s,req} = \frac{M_{Ed}}{0.87 f_{yk} z}$$

Design For Flexural

3) If $M > M_f$, neutral axis in the web

$$\beta_f = 0.167\frac{b_w}{b} + 0.567\frac{h_f}{d}\left(1 - \frac{b_w}{b}\right)\left(1 - \frac{h_f}{2d}\right)$$

$$M_{bal} = \beta_f f_{ck} b d^2$$

$M < M_{bal}$, compression reinforcement is not required

$$A_s = \frac{M + 0.1 f_{ck} b_w d(0.36d - h_f)}{0.87 f_{yk}(d - 0.5h_f)}$$

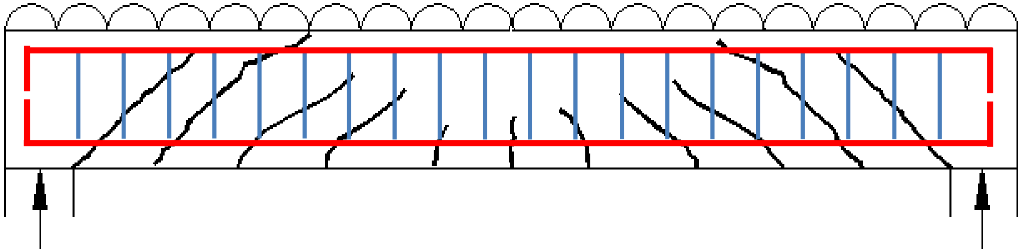
$M > M_{bal}$, compression reinforcement is required

$$A_s' = \frac{(M - M_f)}{0.87 f_{yk}(d - d')}$$

$$A_s = \frac{0.167 f_{ck} b_w d + 0.567 f_{ck} h_f (b - b_w)}{0.87 f_{yk}} + A_s'$$

Design For Shear

- EC2 introduces the strut inclination method for shear capacity checks.
- In this method, the shear is resisted by concrete struts acting in compression and shear reinforcement acting in tension.
- Specified in Cl.6.2.1(8), Cl.6.3(1 & 3), Cl.6.3.(3), Cl.6.2.3(3), Cl.9.2.2(5 & 6).



- where BM is greatest: cracks are caused by bending stress in the tension zone.
- where SF is greatest: cracks are caused by diagonal tension/compression due to complementary shears.

Design For Shear

- Design procedure:
  1) Determine design shear force, V_{Ed}
  2) Determine the concrete strut capacity for *cot θ =1.0* and *cot θ =2.5* (*θ = 22°* and *θ = 45°* respectively)

$$V_{Rd,\max} = \frac{0.36 b_w d f_{ck} \left(1 - f_{ck} / 250\right)}{\cot\theta + \tan\theta}$$

Cl.6.2.3 (3)

If $V_{Ed} > V_{Rd,max}$ *cot θ = 1.0 (θ = 45°)*, redesign the section

If $V_{Ed} < V_{Rd,max}$ *cot θ = 2.5*, use *cot θ = 2.5 (θ = 22°)*, and calculate the shear reinforcement as follows,

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78 f_{yk} d \cot\theta} \quad ; \quad (\cot\theta = 2.5)$$

Design For Shear

If $V_{Rd,max}$ $cot\ \theta = 2.5 < V_{Ed} < V_{Rd,max}\ cot\ \theta = 1.0$

$$\theta = 0.5 \sin^{-1}\left[\frac{V_{Ed}}{0.18 b_w d f_{ck}(1 - f_{ck}/250)}\right]$$

$$\frac{A_{sw}}{s} = \frac{V_{Ed}}{0.78 f_{yk} d \cot\theta} \quad ; \quad s < 0.75d$$

Maximum spacing

3) Calculate the minimum links,

$$\frac{A_{sw}}{s} = \frac{0.08 b_w \sqrt{f_{ck}}}{f_{yk}}$$

Cl.9.2.2(5)

4) Calculate the additional longitudinal tensile force caused by the shear and the additional tension reinforcement,

$$\Delta F_{td} = 0.5 V_{Ed} \cot\theta$$

$$A_s = \Delta F_{td} / 0.87 f_{yk}$$

Deflection

- To control deflection to a maximum of span/250.
- Procedure:

1) Calculate $\rho_o = \sqrt{f_{ck}}\ 10^{-3}$
2) Calculate $\rho = A_{s,req} / bd$
3) Calculate $\rho' = A_{s',req} / bd$
4) Determine K and calculate l/d

Cl.9.2.2(5)

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho} + 3.2\sqrt{f_{ck}}\left(\frac{\rho_o}{\rho} - 1\right)^{3/2}\right] \quad ; \quad \rho \leq \rho_o$$

$$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{ck}}\frac{\rho_o}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho}}\right] \quad ; \quad \rho > \rho_o$$

5) Calculate modification factor, MF_{span} and MF_{area}
6) For adequate deflection control, $(l/d)_{actual} < (l/d)_{allow}$

Deflection

- Factor for structural system can be determined from Table 7.4N:

	Structural System	K	Basic span-effective depth ratio	
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Cracking

- Crack control for beam design can be directly referred to Cl.7.3.
- For a convenient, crack control without direct calculation is preferable, Cl. 7.3.3, Table 7.2N and Table 7.3N.
- Procedure:
  1) Calculate steel stress for limiting crack width, $w_k = 0.3$mm

$$f_s = 435\left[\frac{G_k + 0.3Q_k}{1.35G_k + 1.5Q_k}\right]\left(\frac{A_{s,req}}{A_{s,prov}}\right)$$

Table 7.2N

  2) Determine maximum bar size or bar spacing

Table 7.3N

  3) For adequate crack control,

$$\text{Ø}_{bar,prov} < \text{Ø}_{bar,max}$$

or $S_{prov} < S_{max}$

Cracking

- Maximum bar diameter for crack control (Table 7.2N)

Steel Stress (N/mm2)	Maximum Bar Size (mm)		
	w_k = 0.4mm	w_k = 0.3mm	w_k = 0.2mm
160	40	32	25
200	32	25	16
240	20	16	12
280	16	12	8
320	12	10	6
360	10	8	5
400	8	6	4
450	6	5	-

- Preferable for single layer of reinforcement with adequate clear distance spacing, S_{prov} > max{$Ø_{bar}$; $Ø_{agg}$+ 5mm; 20mm}

Cracking

- Maximum bar spacing for crack control (Table 7.3N)

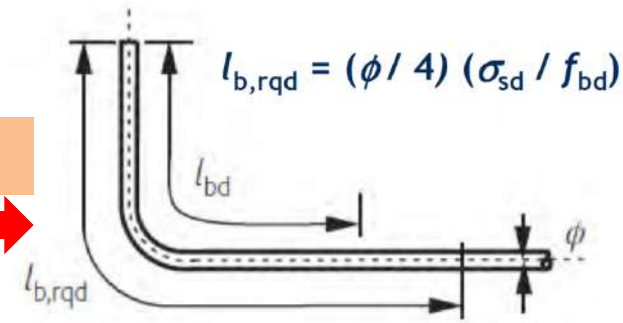
Steel Stress (N/mm2)	Maximum Bar Spacing (mm)		
	w_k = 0.4mm	w_k = 0.3mm	w_k = 0.2mm
160	300	300	200
200	300	250	150
240	250	200	100
280	200	150	50
320	150	100	-
360	100	50	-

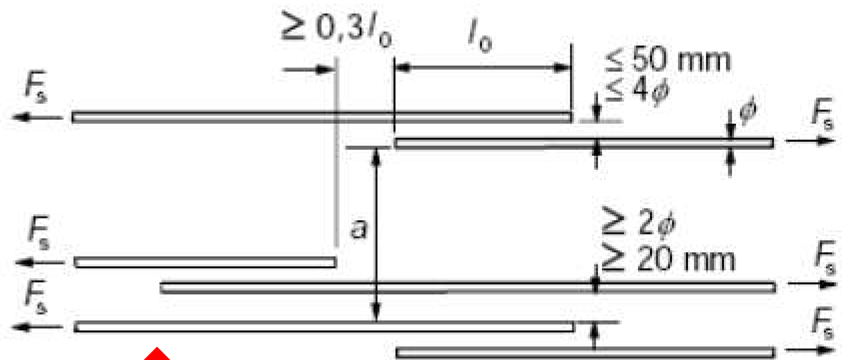
- Preferable for multi-layer of reinforcement as long as attain the maximum diameter reinforcement.
- $S_{prov} < S_{max}$

Detailing

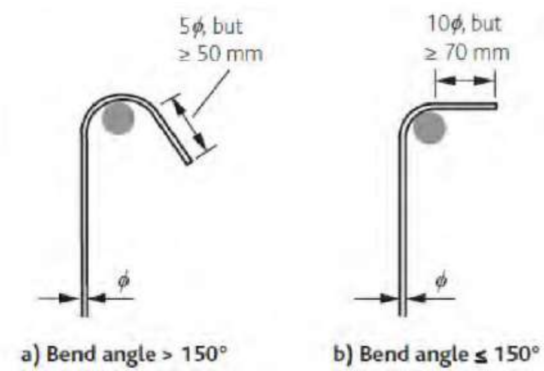
- Detailing of reinforcement in term of curtailment, anchorage and laps must be provided to satisfy the requirement.
  1) Check curtailment , Cl.9.3.1.1(4), 9.2.1.3, Fig.9.2
  2) Check anchorage, Cl.9.3.1.2, 8.4.4, 9.3.1.19(4), 9.2.1.5(1), 9.2.1.5(2)
  3) Check laps, Cl.8.7.3

Anchorage for bar





Laps for bar



Anchorage for link

Detailing

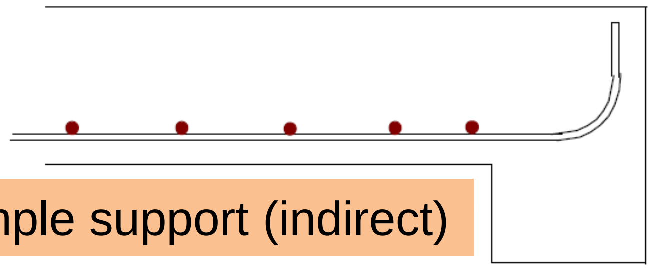
- Simplified rule for anchorage of bottom reinforcement at end supports:
  1) A_s bottom steel reinforcement at support ≥ $0.25A_{s,prov}$ in span.
  2) l_{bd} is required from the line of contact of the support.

$$l_{bd} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5 l_{bd,req} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5(40.3\phi_{bar})$$

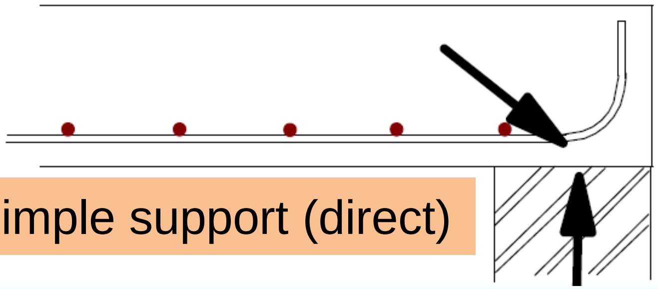
Take conservative value, $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = 1$

$$\alpha_2 = 1.0 - 0.15(c_d - 3\phi)\phi \leq 10$$

Concrete class	C20/25	C28/35	C30/37	C32/40	C35/45	C40/50	C45/55	C50/60
Factor	1.16	0.93	0.89	0.85	0.80	0.73	0.68	0.63



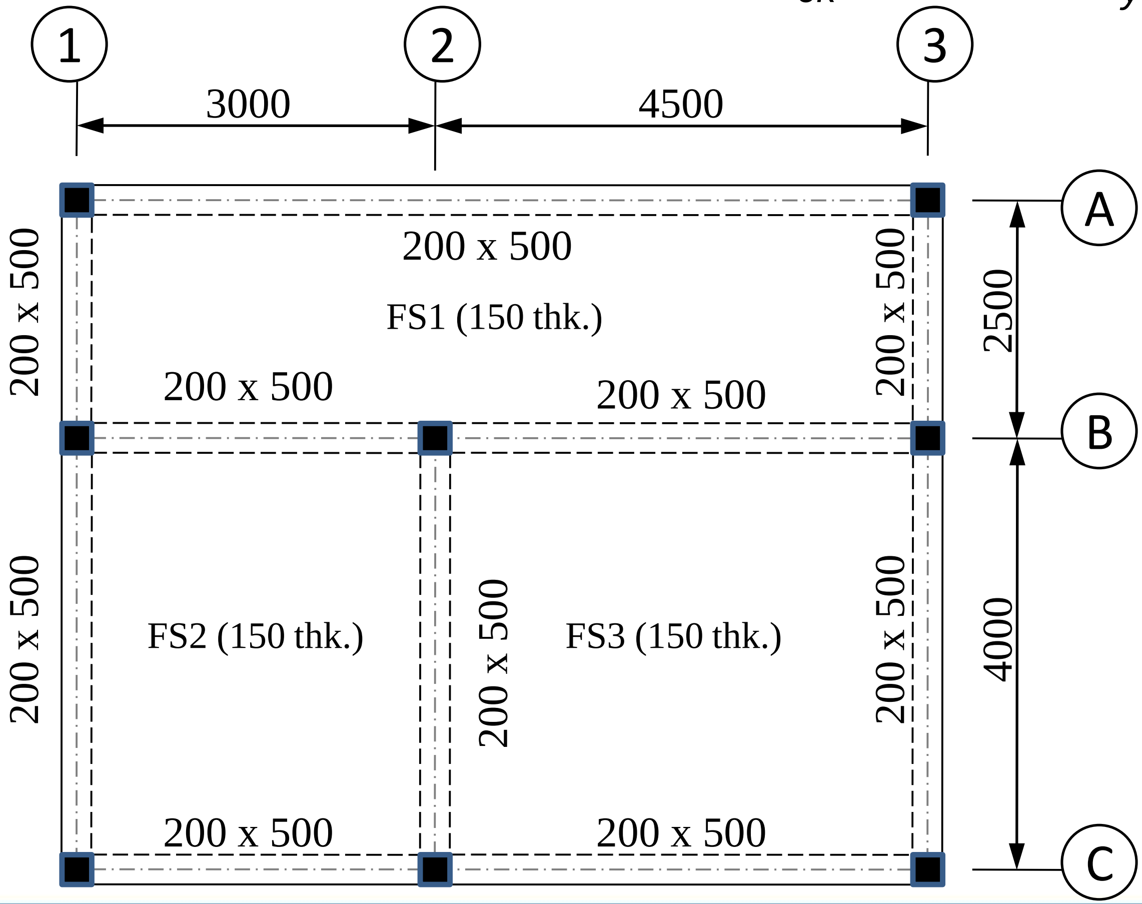
Simple support (indirect)



Simple support (direct)

Tutorial 1

- Design beam 2/B-C. Given the following data: unit weight of concrete = 25kN/m^3; finishes-services = 2.0kN/m^2; variable action (all slabs) = 3.0kN/m^2, f_{ck}= 25MPa, f_{yk}= 500MPa



Action on slab:

$g_{k(s/w)}$ = 3.75 kN/m^2
$g_{k(f\&s)}$ = 2.0 kN/m^2
G_k = 5.75 kN/m^2
Q_k = 3.0 kN/m^2

Type of slab:

FS1, One-way slab
FS2, Two-way slab
FS3, Two-way slab

All unit in mm